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RESEARCH ARTICLE

Cognitive mechanisms for human flocking dynamics

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Abstract
Low-level "adaptive" and higher-level "sophisticated" human reasoning processes have been proposed to play opposing roles in the emergence of unpredictable collective behaviors such as crowd panics, traffic jams, and market bubbles. While adaptive processes are widely recognized drivers of emergent social complexity, complementary theories of sophistication predict that incentives, education, and other inducements to rationality will suppress it. We show in a series of multiplayer laboratory experiments that, rather than suppressing complex social dynamics, sophisticated reasoning processes can drive them. Our experiments elicit an endogenous collective behavior and show that it is driven by the human ability to recursively anticipate the reasoning of others. We identify this behavior, "sophisticated flocking", across three games, the Beauty Contest and the "Mod Game" and "Runway Game". In supporting our argument, we also present evidence for mental models and social norms constraining how players express their higher-level reasoning abilities. By implicating sophisticated recursive reasoning in the kind of complex dynamic that it has been predicted to suppress, we support interdisciplinary perspectives that emergent complexity is typical of even the most intelligent populations and carefully designed social systems.

Keywords Complex game dynamics · Human collective behavior · Behavioral game theory · Cognitive game theory · Iterated reasoning · Adaptive learning

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Introduction

Cultural norms, fads and fashions, and market bubbles and busts are all unpredictable phenomena that can emerge spontaneously from disorganized human collectives [1, 2]. Many mechanisms have been invoked to explain these phenomena, particularly lower-level reasoning processes such as habits, reactions, emotions, biases, reinforcement, and imitation [3–12]. Complex social dynamics, as distinguished from Nash-like equilibria, are characterized by cycles, chaotic attractors, and other non-equilibrium regimes [13]. (By this definition, market equilibrium, the tragedy of the commons, rational herding, and other collective outcomes that basic solution concepts treat as fixed equilibria are not "complex" in our terms.) Because complex systems are difficult to engineer, much less to understand, researchers have put substantial effort into designing systems so that they converge on the most tractable equilibria. One scientific tradition toward this end, going at least back to Keynes [14, 15], holds that increasingly rational behavior can impede the emergence of complex group dynamics by counteracting the influence of our "animal spirits", the adaptive processes that drive group-level complexity [16–21]. Recent theoretical work challenges this narrative by implicating higher-level processes in similarly complex dynamics [22–24]. If sophisticated human-unique reasoning processes can drive group outcomes spontaneously into complex dynamic regimes, then policy interventions that increase motivation, knowledge, or intelligence are not sufficient to increase the simplicity or predictability of real world social dynamics.

Sophisticated flocking in repeated games

Theoretical findings from the nonlinear physics community have shown that simple learning mechanisms foster unexpected emergent dynamics in a wide range of economic games and models [13, 25–28], particularly in games with many players, choices, and repetitions. Prominent among interesting emergent, or endogenous, dynamics is *flocking*, the clustering of human or non-human agents in both their positions and velocities [4, 5]. In the context of repeated games, analogous flocking occurs when many agents converge upon the same series of choices over many rounds of play [13, 29]. Like other collective behaviors, flocking is usually explained in terms of low-level reasoning mechanisms [5, 30, 31], and relevant studies consequently draw heavily from non-human animal collective behavior [4, 32–35].

We focus here on games that elicit a sophisticated flocking collective behavior that, unlike previously observed instances of flocking, is driven by human higher-level reasoning mechanisms. Sophistication is clearest in iterated "what you think I think you think" reasoning [36–39]. This iterated reasoning is an expression of bounded rationality, and it is often invoked for its ability to suppress complex game dynamics associated with flocking [16–19, 40].

In many games, such as those we investigate herein, iterated reasoning makes players' actual choices appear to "hop" discontinuously through a choice space; where a basic adaptive agent advances through neighboring choices sequentially

by best-responding to current state, a sophisticated agent advances more quickly because of its anticipatory best responses to the anticipatory best responses of others. Much of the appeal of games of this type is that researchers can infer players' depths of iteration from the numbers of intermediate strategies they hop over.

We benefitted in this work from online interactive experimental platforms that permitted us to efficiently collect long time series from a large number of interacting groups—from both student and online subject pools—for many repetitions of these somewhat complex games. See details of online deployment in the supplementary information.

Three games

We present experimental results from three economic games, the well-known Beauty Contest and two that we developed, the Mod Game and the Runway Game.

Beauty Contest

In the Beauty Contest, players receive a reward for guessing the number 0–100 that is p (typically, $p = 2/3$) of their group's mean guess [36, 37, 41]. The game motivates participants to try to iteratively anticipate each other's choices, and people who play it perform large hops down through the possible choices. Instructively, a researcher can infer from players' chosen numbers how many iterations they performed, and, by extension, how many iterations they expected from others: a player who responds to suspected choice 50 with 22 is, correctly or not, anticipating that others will be less sophisticated and select 33. The Beauty Contest has a unique pure-strategy Nash equilibrium, which is to say that it has a unique choice—zero—that, when chosen by everyone, leaves no one wishing they had chosen something else. While the Beauty Contest is well known for eliciting sophisticated hopping behaviors, endogenous flocking has not been suspected in the game beyond the superficial clustering of choices over time due to the game's logical structure.

Mod Game

In the Mod Game, n players choose an integer 1–24 (or, more generally, m). Choice 1 dominates ("beats") choice 24 while all other choices x (2–24) beat the choice $x - 1$ exactly below them; players earn one point for making a choice that is exactly one above another player's, zero points otherwise, with all choices linked circularly as along the perimeter of a clock [29, 42] (illustrated in Fig. 1a). This circular structure is directly reminiscent of Rock–Paper–Scissors, which the Mod Game generalizes, and with which it shares the same type of solution: a single "mixed-strategy equilibrium" that players can achieve by selecting choices uniformly randomly. The Mod Game can be made to be zero- or negative-sum by including a penalty of one or more points for being scored upon by the dominating player. Frey and Goldstone found evidence that sophistication and flocking coexist in a positive-sum version of the Mod Game [29].

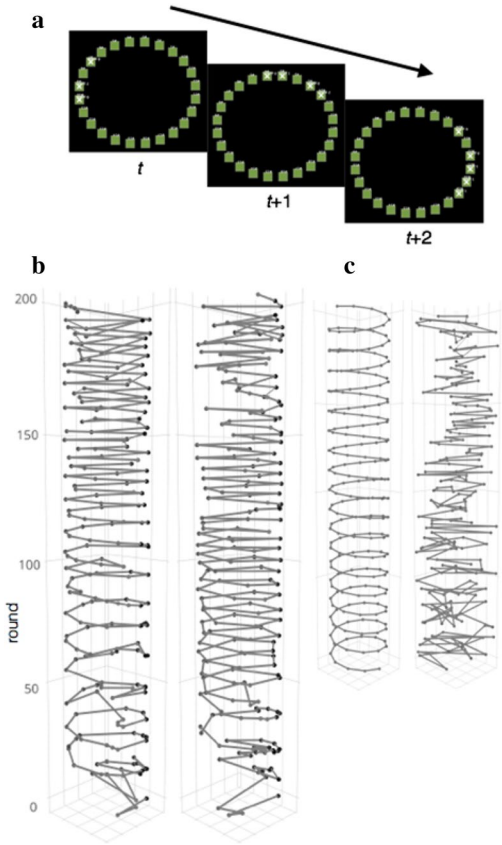


Fig. 1 Cycles in the Mod Game driven by sophisticated flocking. **a** Three rounds of a five-player group in a cyclic flocking pattern. These players are flocking in the sense that they are proceeding together through the same sequence of choices over time. They are sophisticated in the sense that the flock does not advance incrementally through choices, but in larger hops. While reactive players best-respond to current state and advance through choices one at a time, sophisticated players best-respond to the anticipated best responses of others, and advance more quickly in hops, as we observe. **b** 200 rounds of choices 1–24, laid out on a circular strategy space, for players from two groups. **c** Simulated data for comparison: a perfect flock with 200 hops of size 2 and a sequence of 200 randomly generated integers 1–24. The spiral patterns in **b** are consistent with the sophisticated flocking illustrated in the first plot of **c**. Figure S4 plots data from all groups (the data in **b** are from groups 1 and 5)

Runway Game

We developed the Runway Game to narrow the possible explanations for the results of the Mod Game. In both, each of multiple players earns one point for choosing a number precisely one above another's number. However, the choice space of the Runway Game is defined on the real number line, rather than on a circle of integers. A player who enters the number 2.55 will earn a point from anyone who enters 1.55 and can be scored upon by anyone who enters 3.55 (SI). Beyond having an unbounded choice space, the game is unusual because, unrepeated, it has no Nash equilibrium solution (SI). That said, it can gain one with a very small change, the addition of an "outside" choice that guarantees zero earnings. We will see that players are effectively indifferent to this change in the solvability of the basic game, in terms of the behavior patterns that they converge upon.

Three game classes, three types of reasoning

Nash equilibrium is a classic definition of the stability of game outcomes, and gives one model of rationality. But there are many kinds of game equilibria, enough that games can be classified by their solutions and the different methods used to find them [43]. This is important because the formal method that solves a game can not only define its type, but can also suggest the style of reasoning a player should use when playing it. For example, the mathematical method that formally solves the Beauty Contest puts it in the class of "dominance-solvable games". Its presence in this class is what first suggested to researchers that humans might themselves mentally perform iterations analogous to those involved in the mathematics of solving the game [36]. Just as iterated reasoning is expected in games that are solved this way, it is unexpected in games that are solved differently [44–46]. Specifically, in neither the Mod Game nor the Runway Game does formal iterated dominance yield the simple equilibrium solutions that predict the iterated reasoning used by players in the Beauty Contest. The formal approach that yields the Mod Game's "mixed-strategy" Nash equilibrium puts it in a class of games that are considered to elicit random behavior, while the lack of a Nash solution to the Runway Game means that there is not a simple formal prescription for how a rational player should reason about it.

Norms

Because the same mechanisms that drive collective behavior can drive group differences, we follow our analyses of group collective behavior in the above three games with tests of emergent group differences in all three games.

Methods

Measures

In the experiments reported below, all three games were repeated for many rounds, and all players received anonymized but otherwise full information about each previous round's payoff outcomes. In experiments with the Mod Game and Runway Game, we tested both zero- and positive-sum versions, such that a player that was scored upon either did or did not lose a point for every point gained by the scoring player.

Our analyses depend on the concepts of *rate*, through which we identify sophistication, and *choice distance* and *rate distance*, with which we define flocking.

We define a player's rate in round t by comparing their current choice x_t to their previous one. In the Mod Game and Runway Game, rate $x'_t = x_t - x_{t-1}$. Rate greater than one is a proxy for sophistication; in the language of level-k or cognitive hierarchy models this corresponds to $k \geq 2$, reflecting an iterated best response to the anticipated iterated best responses of others [37, 47]. Iterated best response is an established mechanism for the iterative elimination of adjacent series of choices that we describe as hopping.

In a game, flocking occurs when a group's members converge on shared choices and rates (Fig. 1a). It is analogous to flocking in non-human animals, in which many animals converge to common positions and velocities. To capture convergence to the game-theoretic analogues of "position" and "velocity" in the Mod, Runway, and Beauty Contest games, we measure a group's choice distance and rate distance, respectively, as the mean or median difference between all group members' choices and rates within a round. We identify flocking as increasing in a group when both of these distance measures decrease significantly over time (SI: "Measuring rate and hopping"). Under these definitions, sophisticated flocking is flocking with hopping (rate greater than one).

Analyses

Mod Game

We test full and reduced mixed-effect models via ANOVA to estimate the effects of experiment condition, round, group size on three dependent variables: rate (for experimental effects on participants' magnitude of hopping), and choice distance and rate distance (for effects on tightness of flocking). Because we observed substantial across-group differences in behavior, we fit subject ID and group number as random effects (see SI for full specification details). For this and all other analyses, we set a conservative significance threshold of $p < 0.001$, reporting as "marginally significant" all values $0.001 \leq p < 0.05$.

Runway Game

We follow this same statistical strategy for testing the effects of round, group size, and several conditions and controls on hopping and flocking in the Runway Game experiments. Because the experiments tested different manipulations (experiment one: zero sumness, experiment two: pay level), we ran different subsets of these tests on different combinations of the experiments, as appropriate (Table 2; see SI and Tables S5–S7 for full details).

Beauty Contest

Because the behavior we call hopping is well established in the Beauty Contest, we test only for flocking, using the same ANOVA-driven full vs. reduced model strategy. Tests are on linear mixed-effects models with round (normalized to [0, 1]) as the only fixed effect. Due to the small number of rounds and large across-group variability, we defined the distance measures with respect to the median rather than the mean, and modeled behavior with random effects for the means nested within experiment (either Nagel [36] or Ho et al. [37]), experimental condition, group size, and group number.

Aggregate analyses over all games and experiments

We also conduct similar tests of flocking, hopping, and norm formation over all of the games in aggregate. The tests for norm formation are motivated by the large qualitative differences we observed between groups. Because the six experiments we compare varied greatly across several characteristics, we rendered them comparable by articulating some of the tests as comparisons to randomly reshuffled versions of the same experiment's data, by normalizing round numbers across all experiments to the [0, 1] time interval, and by defining "rate" at the group level in terms of a group's previous center of mass (rather than each individual's previous choice; see SI section on measuring rate and hopping). All test results are reported in Table 4.

Results

Despite the formal differences between these games, we robustly elicit the same *sophisticated flocking* behavior in all three.

Mod Game

Eighty-three paid participants in 28 groups of size 2–7 played 100 rounds each of positive-sum and zero-sum versions of the Mod Game (SI. Interface in Fig. S2; experiment conducted January–April 2012). Groups quickly self-organized into a cyclic collective flocking behavior in which individuals "hopped" together around the strategy space (Fig. 1, full data in Fig. 5). Players had a mean hop size of 2.2, meaning that, on average, a player who selected choice 10 in round t tended in round $t+1$ to play choice 12 or 13 (see also Fig. S3). Significant decreases in groups' choice and rate distances over time ($\chi_1^2 = 26$; $\chi_1^2 = 39.2$; both $p < 0.001$; Table 1, also see Fig. 3) reflect a convergence upon shared "positions" and "velocities" through choice space that satisfies our definition of flocking. These distance measures were significantly larger during zero-sum conditions ($\chi_1^2 = 40.2$; $\chi_1^2 = 38.5$; both $p < 0.001$; Table 1), supporting predictions that zero-sum settings will suppress complex dynamic regimes in adaptive agents [26].

The rate of 2.2 is consistent with sophistication. The proximity of players to each other in both choices and rates is consistent with flocking. Sophisticated flocking is inconsistent with prominent theories of game dynamics, which propose that agents with higher-level reasoning should be less susceptible to non-equilibrium flocking in games like the Mod Game [16, 17, 19].

Runway Game

In the first experiment we report with this game, conducted January 2012, 59 players in 17 groups of 2–6 played two 75 round blocks of the game in zero-sum and positive-sum conditions. In the second experiment (positive sum only; 22 groups

of 133 students; two blocks of 50 rounds each paying either 2¢ or 20¢ per point; experiment conducted August–September 2012), we modified the base game to give it a solution, a unique Nash equilibrium in the form of an "outside option" strategy. The SI discusses details of these and an interesting third experiment we ran without independence between groups.

Qualitatively, groups in these Runway experiments tend in initial rounds to cluster on single-digit integers above zero before self-organizing into clusters that climb from round to round through the positive integers (Fig. 2, full data in Fig. 6). Players hop up the strategy space with a median rate of 3 (mode 2; distribution in Fig. S9). They coordinate overwhelmingly on the natural numbers, with only 4.2% of choices being negative or non-integer. They also flock: the median choice distance in a round was seven (mode 2) and the median rate distance was six (mode 1). On these key qualities, behavior in the Runway Game is consistent with sophisticated flocking.

The effect of more rounds of experience is in all Runway experiments significantly larger hops in later rounds and increased flocking (tighter clustering; $\chi_1^2 = 14$;

Table 1 Results over the zero-sum Mod Game experiment

Dependent	Model	Coefficient	*df*	−LL	χ^2	χ^2 *df*	*p*
Rate	Full model		10	56,920			
	Zero sum	−0.526	9	56,920	1.34	1	0.25
	Round	−0.00568	9	56,924	7.67	1	0.0056
	Group size	−0.164	9	56,921	1.41	1	0.24
	Others' rates	**0.0401**	9	56,934	28.6	1	<0.001
	Zero-sum: round	0.00624	9	56,922	3.02	1	0.082
	Zero sum: group size	0.0823	9	56,921	1.32	1	0.25
Choice distance	Full model		9	40,178			
	Intercept	−1.36	8	40,181	8.11	1	0.0044
	Zero sum	**−0.782**	8	40,188	19.9	1	<0.001
	Round	**−0.00410**	8	40,191	26.0	1	<0.001
	Group size	0.173	8	40,179	1.63	1	0.20
	Zero sum: round	**0.00907**	8	40,198	40.2	1	<0.001
	Zero sum: group size	0.0306	8	40,179	1.38	1	0.24
Rate distance	Full model		9	40,484			
	Intercept	−0.668	8	40,485	2.89	1	0.089
	Zero sum	−0.557	8	40,488	9.82	1	0.0017
	Round	**−0.00509**	8	40,503	39.2	1	<0.001
	Group size	0.0586	8	40,484	0.263	1	0.61
	Zero sum: round	**0.00898**	8	40,503	38.5	1	<0.001
	Zero sum: group size	−0.389	8	40,485	2.14	1	0.14

Linear effects on rates, distance in choices, and distance in rates, with random effects for subject and session. Flocking increased with time in the non-zero-sum condition. This table reports effects of χ_1^2 ANOVAs between the full model and reduced versions of it, with one row for each variable tested. Intercept compares not to 0, but to the null distance of 6 that would be expected from purely random play. *Zero sum* is a dummy variable that equals one for choices under the zero-sum condition and zero otherwise. Bold coefficients are significant at $p < 0.001$

$\chi_1^2 = 458$; $\chi_1^2 = 593$; all $p < 0.001$; Table 2). Larger groups and higher pay levels are also associated with larger hops ($\chi_1^2 = 328$, $\chi_1^2 = 17.4$; both $p < 0.001$). Zero-sum versions of the game show no difference in the magnitude of hopping and decreased clustering in choices and rates ($\chi_1^2 = 0.08$, $p = 0.77$; $\chi_1^2 = 91$; $\chi_1^2 = 58$; both $p < 0.001$; Tables S5–S7).

Comparing the results of the two Runway Game experiments, we find that players were indifferent to the presence of a Nash equilibrium, selecting it intentionally in less than 1% of rounds (SI) and exhibiting sophisticated flocking in both versions of the Runway Game.

Beauty Contest

The Beauty Contest is known for eliciting sophisticated iterated reasoning behavior. Since players in the repeated Beauty Contest have multiple rounds to potentially adapt their reasoning depths toward each other, we hypothesized that this game also fosters a previously unsuspected flocking dynamic. We reanalyzed data from two classic studies of the repeated Beauty Contest [36, 37] to examine whether groups in these experiments had formed flocks. The first study administered four rounds with 7 groups of 15–18 people, and the second posed ten rounds with 55 groups of 3–7.

It is established that choices in the Beauty Contest become closer to each other as they approach zero: from choice 50, iterating to depths one and two yields choices (33 and 22) that differ by 11, while starting the same processes from choice 5 yields outcomes that differ by just 1.1. This is not the endogenous flocking we are interested in identifying; it is a natural artifact of the structure of the game. We propose choice and rate clustering that occurs over and above this spurious type. To measure clustering due specifically to endogenous flock formation due to mutual adjustment of strategies from round to round, we calculated group distance by round after normalizing choices and rates in each round relative to the benchmark set by the previous round's winning choice (SI). We find evidence for flocking over both experiments (via a significant increase in both choice and rate clustering; Fig. 3, top 2 rows; $\chi_1^2 = 25.3$; $\chi_1^2 = 22.5$; both $p < 0.001$; Table 3).

Since group-level clustering of choices should occur in other environments where participants are trying to match the "position" and "velocity" of a group-level variable, this evidence for emergent flocking in classic Beauty Contest data gives additional support for Nagel's original prediction that participants adjust their depth of reasoning from round to round based on feedback from the previous round. This is a route by which higher-level reasoning can drive complex collective dynamics.

Cross-game evidence for emergent flocking

Figure 3 compares six experiments on three games—three experiments reported here, and three reanalyses—to give evidence that flocks emerge in multiple classes of games. A linear model over all three games finds increased flocking—lower choice distance and rate distance—with time ($\chi_1^2 = 85.8$; $\chi_1^2 = 99.9$; both $p < 0.001$;

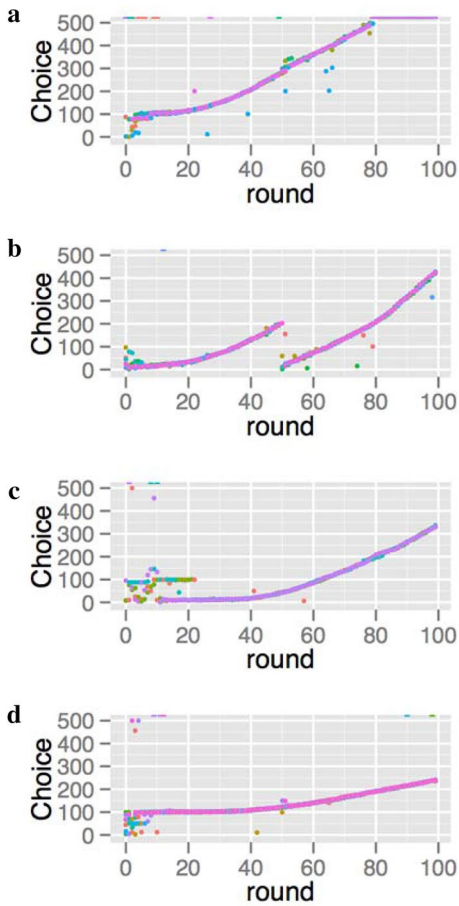


Fig. 2 Sophisticated flocking in four group sessions of the Runway Game. Plots show raw choices over 100 rounds. Each color represents a different group member. The apparent proliferation of points in initial rounds reflects disorganized behavior before groups have formed flocks, which appear in each plot as a line. The slopes of all lines are greater than 1, which can occur only when players are best-responding to the recursively anticipated best responses of others. Furthermore, the different slopes of each panel suggest differing norms for how players come to expect each other to reason (Fig. 4). Choices above 500 are truncated. These sessions used a two-block design, with each block lasting 50 rounds (this explains the "reset" in **b**). **a**–**d** Groups 70–73 of Fig. 6, showing all sessions

Table 2 Key results over all Runway Game experiments

Dependent	Model	Coefficient	*df*	−LL	χ^2	χ^2 *df*	*p*
Rate	Full model		7	91,555			
	Round	**0.0142**	6	91,562	14.2	1	< 0.001
	Group size	**0.4**	6	91,712	328	1	< 0.001
Choice distance	Full model		6	69,537			
	Round	**−0.0082**	5	69,766	458	1	< 0.001
	Group size	0.003	5	69,537	0.0226	1	0.88
Rate distance	Full model		6	75,864			
	Round	**−0.012**	5	76,164	593	1	< 0.001
	Group size	−0.042	5	75,864	0.29	1	0.59

Models of rate, choice distance, and rate distance. All three dependents increased significantly with round. This table reports effects of χ^2_1 ANOVAs between the full model and reduced versions of it, with one row for each variable tested. This table aggregates the first two rows of Tables S5–S7; see them for full details and within-experiment tests. Bold coefficients are significant at $p < 0.001$

Table 4). Across experiments, we see that flocks emerge with or without high pay levels, with or without Nash equilibria, and also that they are robust to group size, subject pool, and both within- and between-session experience (SI).

Cross-game evidence for emergent within-group norms

We also tested for cross-group differences in how groups flocked and expressed iterated reasoning. To test for group-specific norms, we compared groups in our experiment to synthetic groups recalculated from a version of the dataset in which participants and their series of choices were randomly reassigned to different groups (SI: "Norm formation"). We find (1) that participants' apparent depths of reasoning (median rate) show great variability across groups, but remain similar within groups (Fig. 4, Table 4), (2) that an individual's rate predicts the median rate of the rest of their group ($\chi_1^2 = 16.8$; $p < 0.001$), (3) that the variation in members' rates decreases with time ($\chi_1^2 = 171$; $p < 0.001$), and (4) that these patterns disappear in randomly reshuffled groups (Table 4). Experiments that do not show evidence of norm formation in sophistication depth are the zero-sum Mod Game (perhaps because zero sumness should suppress flocking [26]) and the four-round Beauty Contest (which probably converges too quickly for norms to form). We conclude that groups develop their own customary limits on the number of strategic iterations that members expect each other to perform. The implicit social norm on cognitive sophistication develops in parallel with the sophisticated flocking collective behavior, which itself seems to leave fundamental parameters to endogenous processes. Even basic aspects of observed collective behaviors seem to vary by experimental group, such that these games seem to be complex enough to foster several qualitatively different collective behaviors (Fig. 6, also Fig. 5): does rate stay constant or increase steadily? Do groups intermittently "reset" to zero? Does sophisticated flocking emerge at all, or do participants collectively come to favor some other group pattern?

Discussion

One flocking behavior in all games

Players in the Mod Game, Runway Game, and Beauty Contest self-organize a common "sophisticated flocking" behavior, even though each game affords different formal solution concepts and should evoke different varieties of strategic reasoning.

This argument is clearest for the games in their single-shot forms, but it is true of their repeated versions as well. It is true that game theory's folk theorems introduce simple conditions that create in repeated games infinitely many Nash equilibrium strategies [48–53]. By permitting arbitrarily many novel collective behaviors, these feasibility theorems may seem to trivialize any discoveries of interesting dynamics, but folk theorems offer no predictions for the specific form that repeated game dynamics will take, and no account for why the same group dynamic might appear

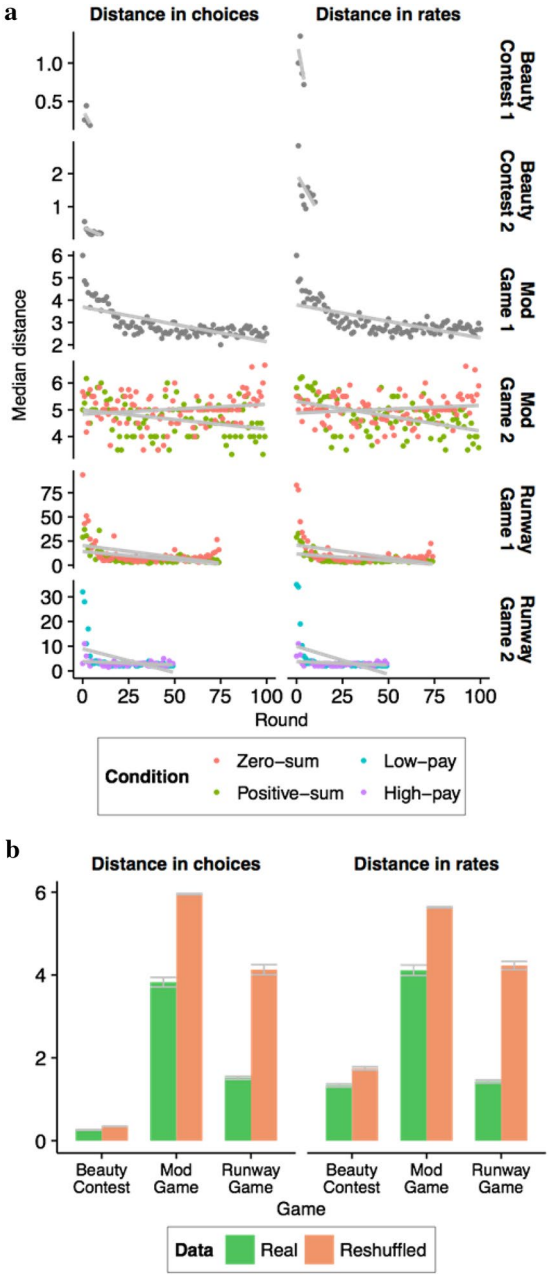


Fig. 3 Cross-game evidence for flocking. We define flocking as within-group, within-round convergence on both choices (first column of both panels) and rates (or hops; second column). In the scatter plots of **a**, each point represents a distance for a group in a round. These plots show that the distances between group members' choices and rates decrease over time, particularly in the initial rounds of play. Beauty Contests are limited in how many times they can be repeated, but endogenous flocking is clear in even these short time series, and even after controlling for the superficial flocking-like behavior reported in the original work on the Beauty Contest [36]. The other experiments vary in their number of rounds because we discovered, as these paradigms matured, that the collective behavior of interest needed only tens of rounds to emerge reliably. The first three rows of **a** (in gray) illustrate the flocking we identify in previously published data [29, 36, 37], and the last three plots (in color) show the data reported herein. Flocking increases with time (negative slopes) except in zero-sum conditions of the Mod Game and Runway Game. **b** Extent of flocking by comparing observed flocking to that in synthetic reshuffled datasets that randomly reassigned participants across groups. Bars show standard errors

independently in games of very different types. Folk theorems fail to provide a positive theory of repeated games. Furthermore, any failure of a game's single-shot properties to influence modeling of its repeated version is important to be able to explain. In this sense, established theory cannot account for our findings.

Table 3 Sophisticated flocking in two classic Beauty Contest experiments

Dependent	Tested variable	Coefficient	Model df	−LL	χ^2	χ^2 *df*	*p*
Choice distance	Round	**−1.126**	4	9985	25.3	1	<0.001
Rate distance	Round	**−0.933**	4	9517	22.5	1	<0.001

Results of mixed-effects models for the two indicators of flocking over two classic Beauty Contest experiments [36, 37]. We calculated the distances of choices from each other, normalized relative to the benchmark set by the previous round's winning choice. We find that flocking increases with time over both experiments. Each row reports an ANOVA comparing the full model to a version without a variable for round. −LL gives the negative log-likelihood after removing this variable from the full model. Bold coefficients were significant below $p < 0.001$

Table 4 Analysis of sophisticated flocking and endogenous norm formation across all three games

Dependent	Tested variable	Coefficient	*df*	−LL	χ^2	χ^2 *df*	*p*
Choice distance	Round	**−0.521**	8	90,521	85.8	1	<0.001
	Reshuffled	1.04	8	90,480	3.91	1	0.048
	Reshuffled: round	**0.547**	8	90,508	60.1	1	<0.001
Rate distance	Round	**−0.550**	8	89,672	99.9	1	<0.001
	Reshuffled	1.16	8	89,625	4.98	1	0.026
	Reshuffled: round	**0.564**	8	89,656	67.0	1	<0.001
Standard deviation of rates	Round	**−1.21**	8	111,411	171	1	<0.001
	Reshuffled	0.452	8	111,327	3.31	1	0.069
	Reshuffled: round	**0.400**	8	111,331	11.9	1	<0.001
Median rate of rest of group	Round	−0.176	10	131,509	1.25	1	0.26
	Reshuffled	**5.44**	10	132,369	1720	1	<0.001
	Rate	**0.860**	10	131,517	16.8	1	<0.001
	Reshuffled: rate	**−0.591**	10	132,859	2700	1	<0.001
	Round: rate	−0.044	10	131,513	9.58	1	0.0020

Results of mixed-effects models for four dependent variables. The first two establish that flocking increases with time over all games, and that this effect of time is reversed for reshuffled data. The second two establish emergent norm formation in all three games, and also show the reshuffling eliminates these effects. Each row compares the full model named in the first column to a version without the tested variable. −LL gives the negative log-likelihood after removing this variable from the full model, and the first *p* value column reports the significance of the change in likelihood after a χ^2_1 test. Bold coefficients are those that were significant below $p < 0.001$

Hopping due to iterated reasoning

The most likely explanation for hopping is that players come to conceive of these games as evoking sophisticated oneupmanship via iterated best response. There are four reasons to support iterated reasoning as the explanation for hopping: (1) the most common rates in all three games were consistent with the one to three steps of iterated reasoning common in the literature [47]; (2) iterated reasoning provides a more parsimonious and psychological explanation than alternative theories (SI); (3)

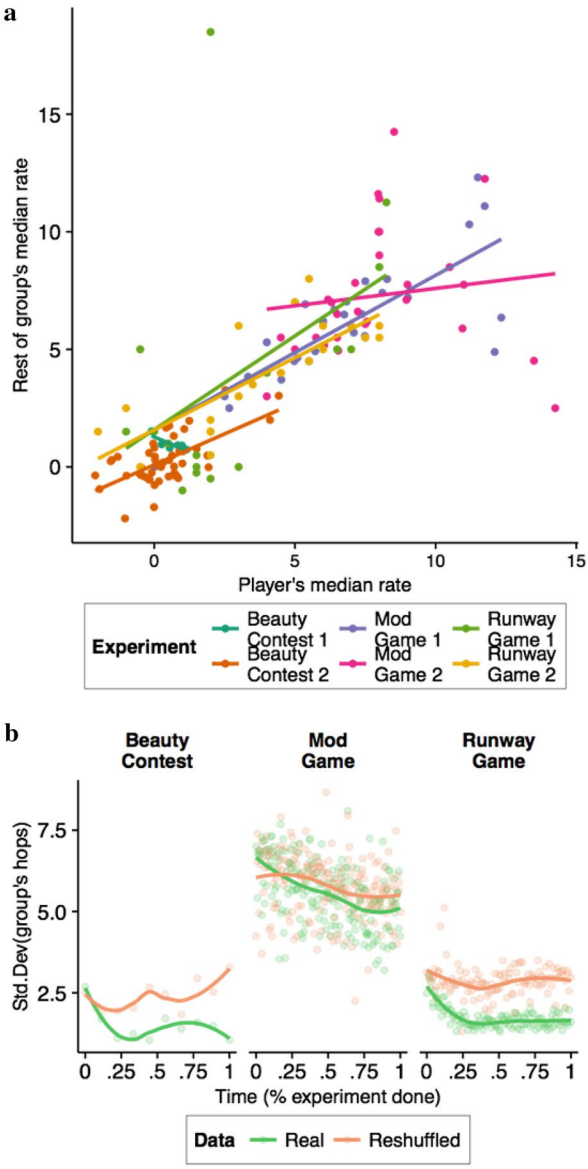


Fig. 4 Emergent group-level norms. Both panels present evidence that groups developed varying norms for the shared depth of recursive reasoning that their members exhibited. Greater hop sizes correspond to more deeply recursive strategic reasoning. **a** Median hop sizes exhibit both within-group homogeneity (slopes are ~ 1) and cross-group heterogeneity (axes have large domain and range). **b** The variances in groups' hops decrease (green lines), particularly in the initial rounds of each game, and remain below that of groups that have been randomly reshuffled (orange)

our findings in the Mod Game and Runway Game inspired our successful prediction of flocking in the Beauty Contest, in which iterated reasoning is already well established as the cause of hopping; and (4) informally, participants often mentioned after sessions that they had used iterated reasoning to inform their choices.

The co-occurrence of sophistication and flocking is a challenge for theories of game learning that treat iterated reasoning as a solution to non-equilibrium dynamics. One particularly challenging result for such theories is the finding that, over

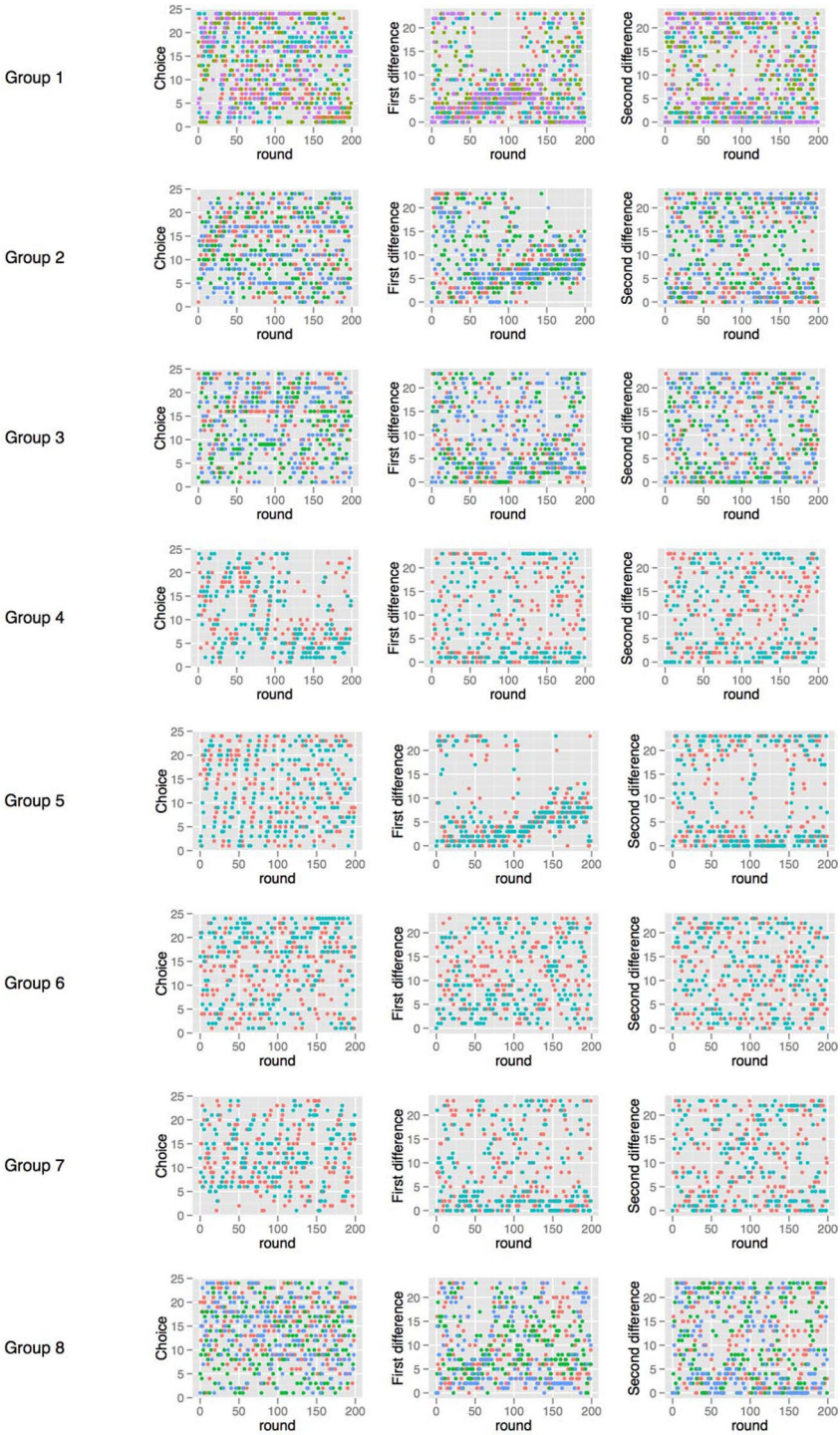
Fig. 5 Mod Game experiment: raw choices, rates, and accelerations over time, by experimental session, ▶ show the emergence and persistence of sophisticated flocking. Each row of three graphs plots the time course of an experimental group, with choices (1–24) leftmost, rates (the difference between a participant's previous and current choice) in the middle, and accelerations (the difference from their preceding rate) on the right. Hopping is most clearly evident in the choice and rate plots. Steep diagonal striations in a choice plot reveal a pattern of participants' choices climbing up to 24, and then wrapping back around to the bottom of the choice space. Rate plots show that the steepness of this climbing—the magnitude of hopping through the strategy space—was mostly between 1 and 4. Flocking is most clearly evident in the choice and acceleration plots. Where striations are most evident, participants are hopping through the strategy space together. The rightmost acceleration plots showing that adjustments to rate remained just above or below zero, as players track their group (keep in mind that, on a circle, values up near 24 are immediately "below" zero). Compare this data to the much stronger flocking evident in the data from the entirely positive-sum "Mod Game 1" of Fig. 3a: http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0056416#s5. While different groups were qualitatively observed to converge on different dynamic regimes (compare, for example, groups 5, 15, 19, 20, and 25 and various small groups whose behavior is difficult to distinguish from random), we focus mainly on the collective behavior we observed to be most common and apparent in the aggregated data: sophisticated flocking, most obviously evident in groups 1, 5, 11, 13, 14, 21, 23, and 25. Within each plot, different color points represent different participants. Group numbers are in terms of the Experiment ID list in Table S1, which also labels sessions by condition. See Fig. 6 for comparable patterns from Runway Games

time, players in both the Mod Game and the Runway Game show simultaneous increases in both depth of reasoning and tightness of clustering.

Despite these arguments in favor of iterated reasoning in the Mod Game and Runway Game, there is some evidence that undermine a straightforward explanation for hopping in terms of iterated reasoning. Most importantly, there is little theoretical justification for invoking iterated reasoning in either the Mod Game or Runway Game, since the analytic procedure upon which the psychology of iterated reasoning is formally grounded, the iterated deletion of dominated strategies, does not yield rational benchmark solutions in either. Second, if players are constantly trying to one-up each other, ad infinitum, it would seem that a rational player's rates would not stabilize, but diverge toward infinite iterations over time. Additionally, players in some groups exhibit rates of ten or more. This is hard to explain in terms of a faculty for iterated reasoning that is only rarely observed to go beyond depth three. Last, despite the supposedly complex deliberative process required for ever deeper recursive reasoning, participants respond in the same unrealistically short amounts of time (0–10 s), regardless of how many iterations they seemed to have performed.

An emergent mental model

These subtle problems in the ability of iterated reasoning to explain behavior in the Mod Game and Runway Game may be resolved if we introduce a distinction between iterated reasoning as a cognitive process and iterated reasoning as a "mental model". A mental model is the conceptualization of a scenario within which one decides how to reason about it [54, 55]. Suppose that a simple decision heuristic such as "increment previous choice by n" can produce an "unsophisticated" hopping behavior superficially consistent with fully deliberative iterated "what you think I think you think I think" reasoning. If all players think of each other as applying the deliberative process, but each in fact uses the cognitively cheap heuristic, then


Group 1
Group 2
Group 3
Group 4
Group 5
Group 6
Group 7
Group 8
Choice
First difference
Second difference
round
0
5
10
15
20
25
50
100
150
200

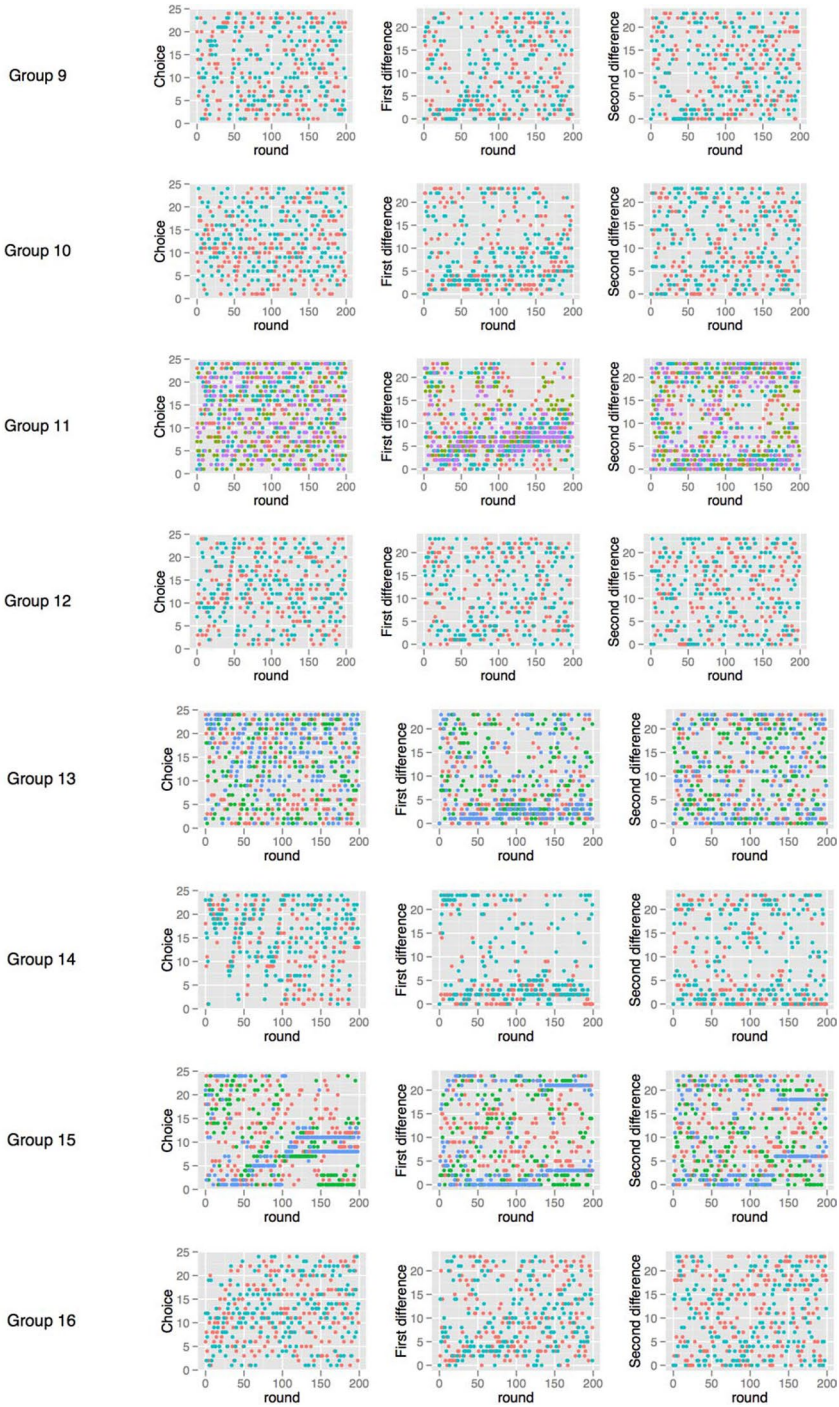
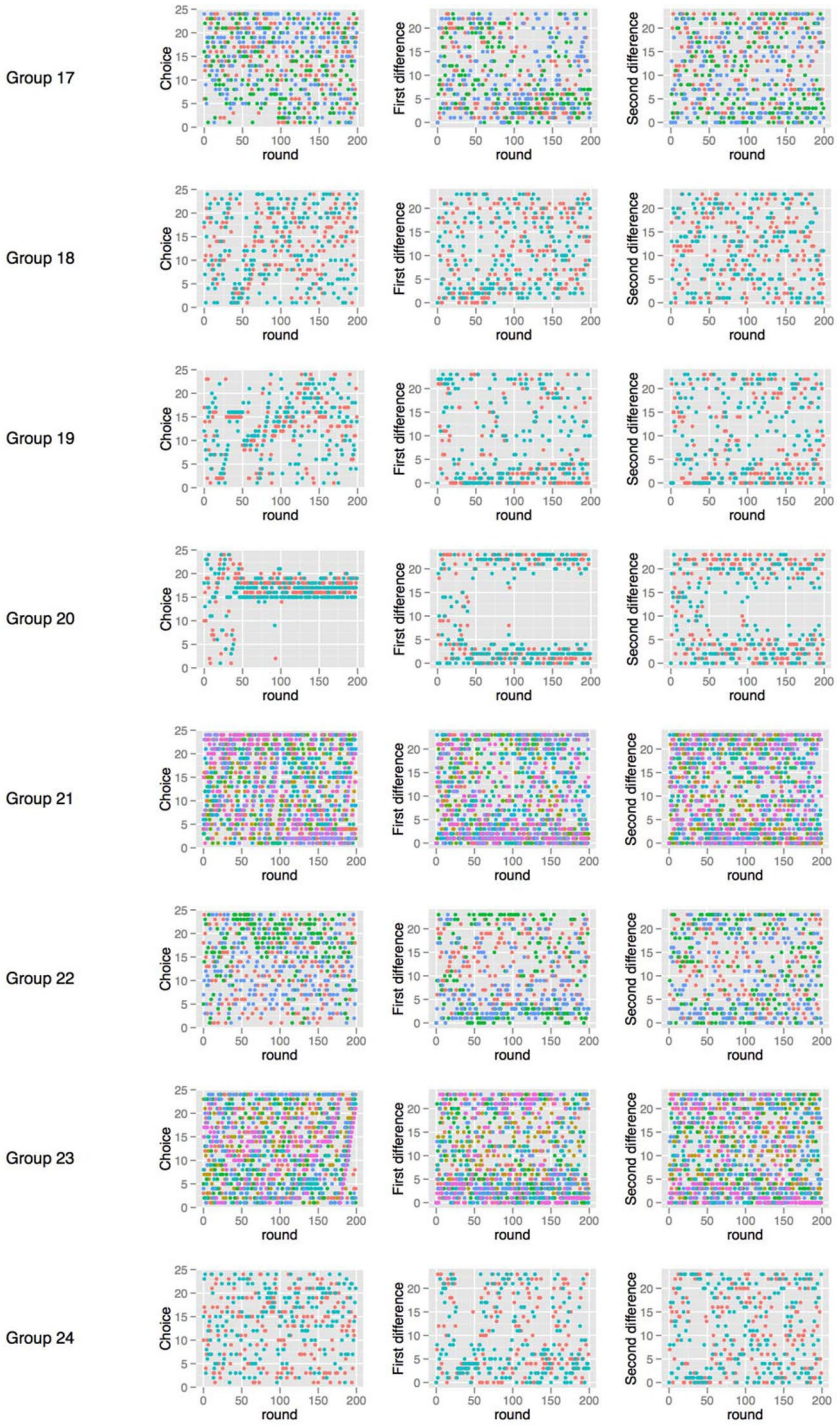


Fig. 5 (continued)



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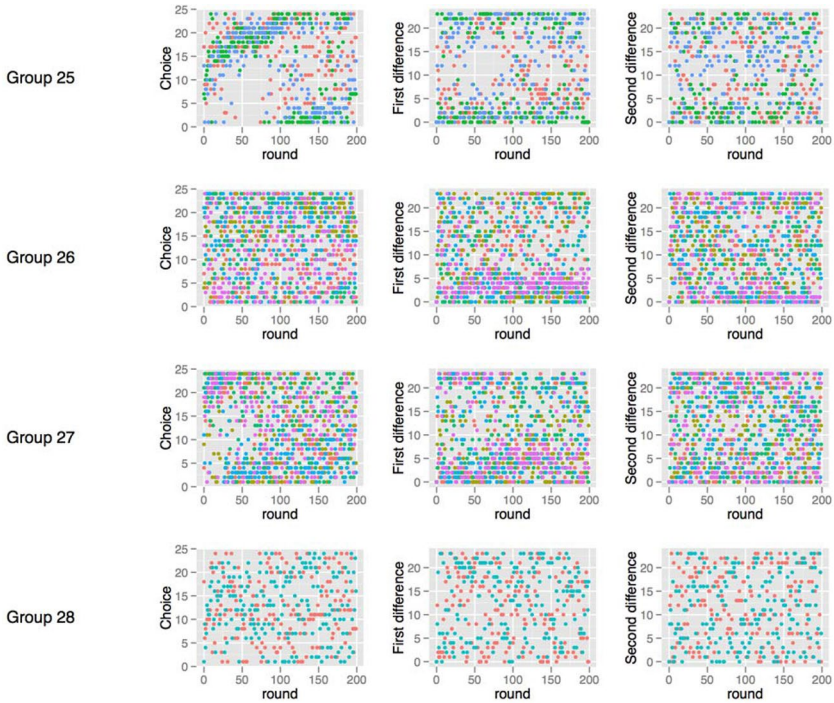
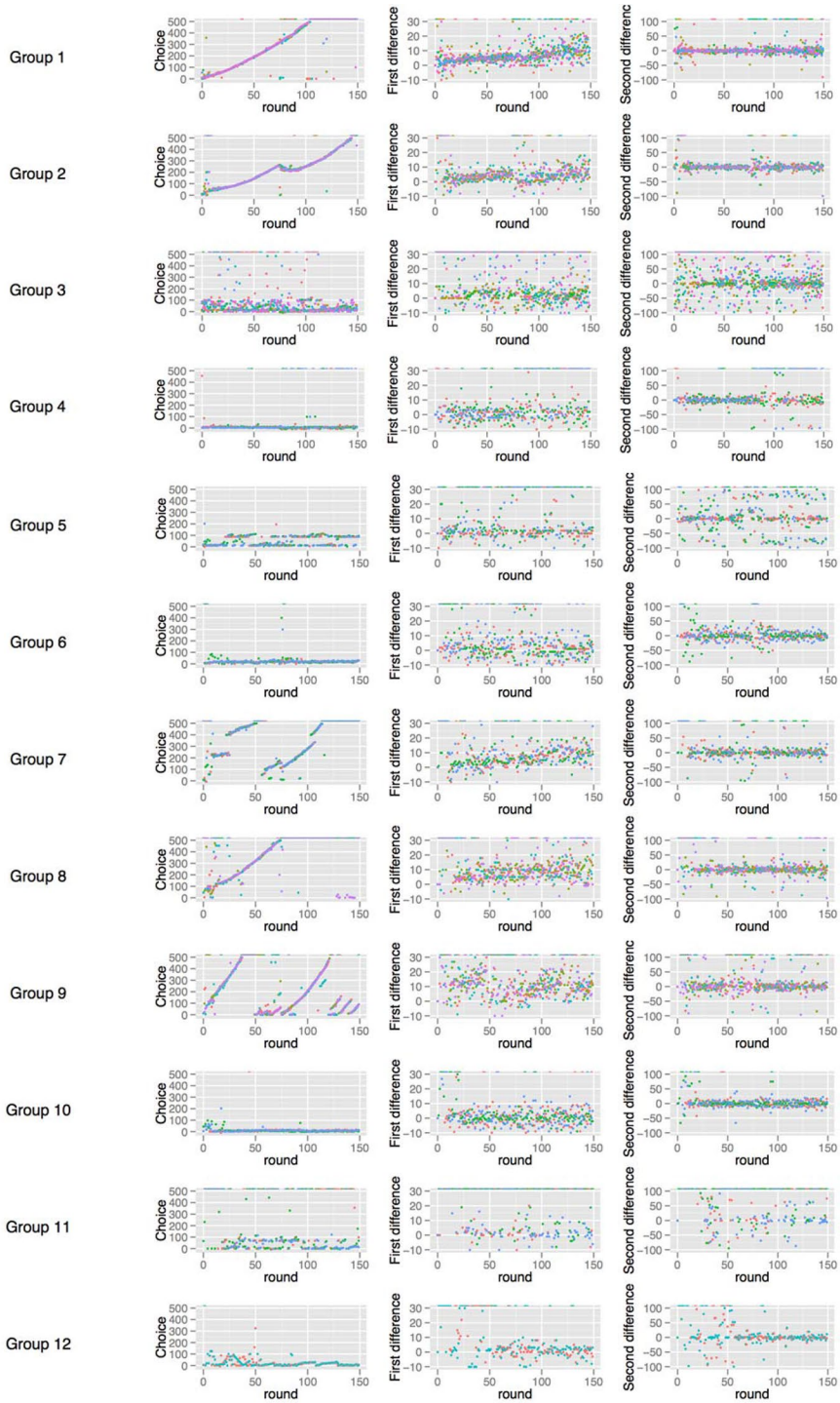


Fig. 5 (continued)

Fig. 6 Runway Game experiments one and two: raw choices, rates, and accelerations over time, by experimental session, show the emergence and persistence of sophisticated flocking. Each row of three graphs plots the time course of each experimental groups' behavior, with choices in the left column, rates in the middle, and accelerations on the right. Flocking is clearly apparent in the emergence of solid lines in the choice plot; participants could select any real number, but within the first rounds they converged upon a pattern of selecting numbers near to each other, and climbing so tightly together through the choice space as to paint clear lines in the plot. As in Fig. 5 showing the Mod Game data, rates are mostly a bit above zero for the whole experiment, and accelerations cluster above and below zero. Hopping is apparent in the steepness of the slope in the leftmost plots, and in how rates clustering steadily above 0. Despite the consistency of sophisticated flocking across sessions, groups vary in several details of how they manifest that behavior, with some apparently maintaining constant rate up the integers (such as groups 63 and 64), some continuously accelerating (groups 2, 62, and 72), some accelerating for a period before rates stabilize (65, 67, 70), some "resetting" choices back to zero intermittently (9, 13, 66, 82)—especially at the halfway condition boundaries (79, 74, 71)—some resetting their rate but not their choices (2, 14, and 82), and several persisting in a regime of flocking without hopping, in which choices remain within a bounded range for the duration of the experiment (such as 3–6 and 10–12), a behavior that might reflect some participants conceptualizing the game as one in which it is profitable to "lay a trap" by making one choice repeatedly, occasionally incrementing by two. Within each plot, different color points represent different participants. To aid visualization, choices were truncated to within [0, 500], rates to [− 10, 30], and accelerations to [− 100, 100]. Points outside any of these bounds are plotted at the limit. Group numbers are in terms of the group list in Table S3, with group sessions 1–17 from experiment one, and 62–83 from experiment two


Group 1
Group 2
Group 3
Group 4
Group 5
Group 6
Group 7
Group 8
Group 9
Group 10
Group 11
Group 12
Choice
First difference
Second differenc
round

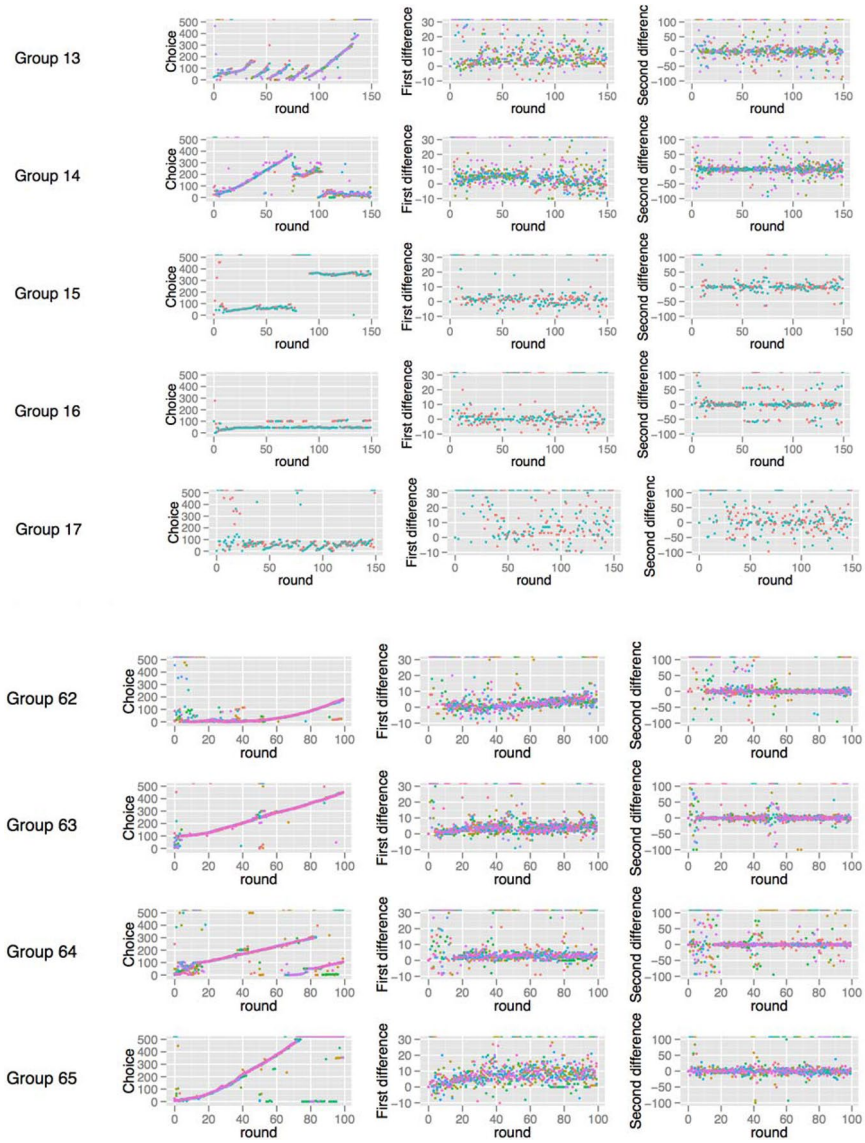


Fig. 6 (continued)

iterated reasoning is no longer applied by anyone as a cognitive procedure for generating behavior, but it continues to be applied by everyone as a way of understanding how others play the game.

The idea of mental models may help explain how sophisticated flocking emerged in all three games: despite their underlying formal differences, players somehow conceptualized these three games similarly enough to contribute to the same collective behavior across them. While a weaker, "mental model-free" claim might

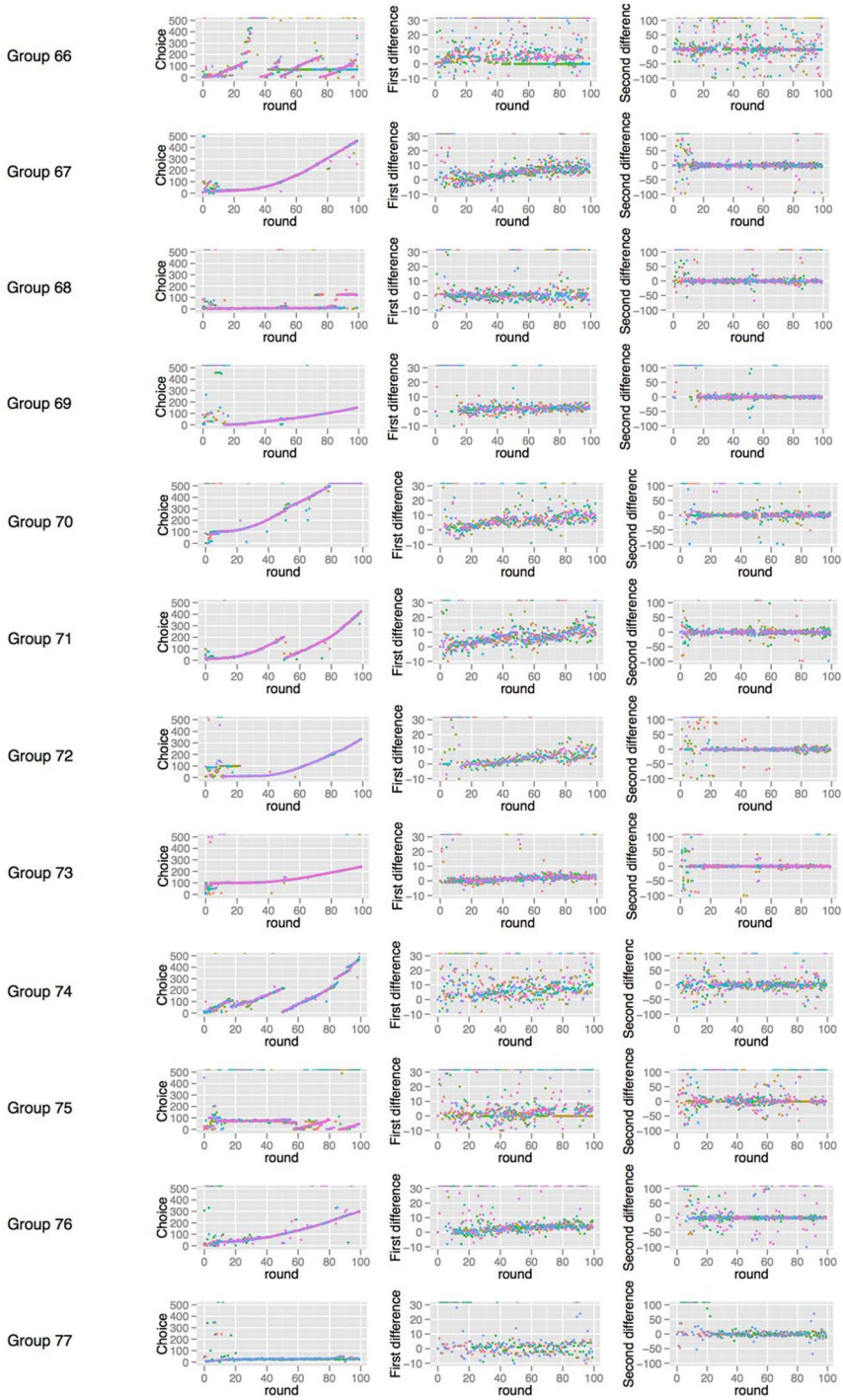


Fig. 6 (continued)

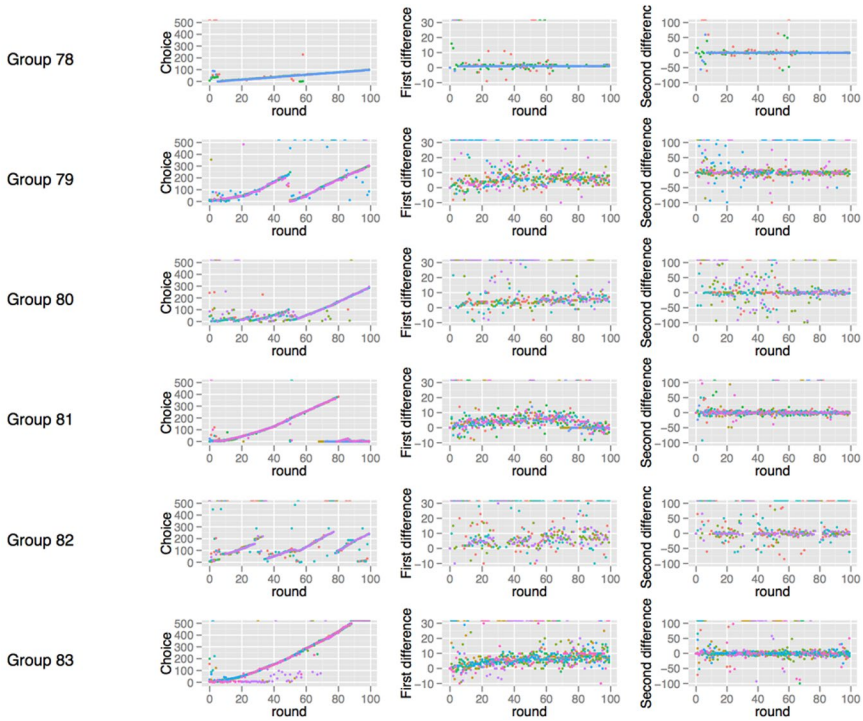


Fig. 6 (continued)

suffice to explain the emergence of flocking in any one of the games considered on their own, the fact that all three engender the same very specific collective behavior seems to imply some unexplained similarity between them via a conceptual level of game understanding that intercedes between game theory and game behavior. In the extreme case of the Runway Game, participants self-organize the same behavior regardless of whether or not the game even has a Nash solution. This insensitivity of players to basic game properties challenges the relevance of elementary game-theoretic concepts to more complex games [56].

An emergent norm on depth of reasoning

The connections we find between sophistication and collective behavior leave the door open for further interactions between them. For example, if players are repeatedly trying to one-up each other, why do their rates not diverge to infinity; why does each group stay within a relatively narrow range of rates?

In games that elicit strategic iterated reasoning, players usually perform only zero to three iterations [47]. Researchers have long debated whether these limited depths are due to players' own cognitive limits, or instead to the limits they expect of others [36, 41, 57–60]. We offer a third possibility for when games are repeated, one that is

only beginning to receive attention [61–63]: expressed depths of iterated reasoning are a social norm that emerges, for each group, as a result of groupmates' previous patterns of mutual adaptation to each other. For example, given two players who adapt their depth of iterated reasoning toward each other over multiple rounds, with one initially expressing a depth of 1 and the other a depth of 5, a few rounds of experience will find them both expressing depths of 3. Two symptoms of the existence of a group-level norm on hop size should be a correlation between a player's rate and their group's rate, and also decreasing variance in group members' rates with time, as compared to when the data is shuffled to randomly reassign participants across groups. We observe both (Fig. 4).

If, as we show, different groups converge on different customary rates of flocking, then an individual's depth of iterated reasoning may not reflect their own abilities, or the abilities they expect of others, but an endogenous norm for how far ahead everyone thinks everyone should try to think. From this perspective, sophistication does not merely fail to suppress complex group dynamics; its expression is affected by them.

Conclusion

In three different repeated economic games, laboratory groups produce sophisticated flocking, a collective behavior driven by human higher-order strategic reasoning. Our tests demonstrate the robustness of this behavior over high and low payoffs, over multiple populations, and with very experienced players. The emergence of one collective behavior in all three games demonstrates that a conceptual similarity between games can override important formal differences in determining how groups ultimately play a game. Because of the evidence that emergent social dynamics limit iterated reasoning (rather than the other way around), these games show that sophisticated human reasoning processes may be just as likely to drive the complex, often pathological, social dynamics that we usually attribute to reactive, emotional, non-deliberative reasoning. In other words, human intelligence may as likely increase as decrease the complexity and unpredictability of social and economic outcomes.

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References

1. Bikhchandani, S., Hirshleifer, D., & Welch, I. (1998). Learning from the behavior of others: Conformity, fads, and informational cascades. *Journal of Economic Perspectives, 12,* 151–170.
2. Schelling, T. C. (1978). *Micromotives and macrobehavior*. New York: W. W. Norton & Company.

3. Castellano, C., Fortunato, S., & Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of Modern Physics, 81,* 591–646. https://doi.org/10.1103/revmodphys.81.591.
4. Dyer, J., Ioannou, C., Morrell, L., et al. (2008). Consensus decision making in human crowds. *Animal Behaviour, 75,* 461–470.
5. Raafat, R. M., Chater, N., & Frith, C. (2009). Herding in humans. *Trends in Cognitive Sciences, 13,* 420–428. https://doi.org/10.1016/j.tics.2009.08.002.
6. Fowler, J. H., & Christakis, N. A. (2010). Cooperative behavior cascades in human social networks. *Proceedings of the National Academy of Sciences, 107,* 5334–5338. https://doi.org/10.1073/pnas.0913149107.
7. Lux, T. (1999). Scaling and criticality in a stochastic multi-agent model of a financial market. *Nature, 397,* 498–500.
8. Helbing, D., Farkas, I., & Vicsek, T. (2000). Simulating dynamical features of escape panic. *Nature, 407,* 487–490.
9. Kerr, B., Riley, M. A., Feldman, M. W., & Bohannan, B. J. M. (2002). Local dispersal and interactions promote coexistence in a real-life game of rock–paper–scissors. *Nature, 418,* 171–174.
10. Semmann, D., Krambeck, H. J., & Milinski, M. (2003). Volunteering leads to rock–paper–scissors dynamics in a public goods game. *Nature, 425,* 390–393.
11. Kirkup, B. C., & Riley, M. A. (2004). Antibiotic-mediated antagonism leads to a bacterial game of rock–paper–scissors in vivo. *Nature, 428,* 412–414.
12. Nowak, M. A. (2006). Five rules for the evolution of cooperation. *Science, 314,* 1560–1563. https://doi.org/10.1126/science.1133755.
13. Sato, Y., Akiyama, E., & Crutchfield, J. P. (2005). Stability and diversity in collective adaptation. *Physica D: Nonlinear Phenomena, 210,* 21–57. https://doi.org/10.1016/j.physd.2005.06.031.
14. Barnett, V. (2015). Keynes and the psychology of economic behavior: From stout and sully to the general theory. *History of Political Economy, 47,* 307–334.
15. Koppl, R. (1991). Retrospectives: Animal spirits. *The Journal of Economic Perspectives, 5,* 203–210. https://doi.org/10.1257/jep.5.3.203.
16. Selten, R. (1991). Anticipatory learning in two-person games. In R. Selten (Ed.), *Game equilibrium models: Evolution and game dynamics* (p. 98). Berlin: Springer.
17. Bloomfield, R. (1994). Learning a mixed strategy equilibrium in the laboratory. *Journal of Economic Behavior and Organization, 25,* 411–436.
18. Stahl, D. O. (2000). Rule learning in symmetric normal-form games: Theory and evidence. *Games and Economic Behavior, 32,* 105–138.
19. Tang, F. F. (2001). Anticipatory learning in two-person games: Some experimental results. *Journal of Economic Behavior and Organization, 44,* 221–232.
20. Camerer, C. F. (2003). Chapter 6: Learning. *Behavioral game theory: Experiments in strategic interaction* (pp. 265–335). Princeton: Princeton University Press.
21. Camerer, C. F., & Fehr, E. (2006). When does "Economic Man" dominate social behavior? *Science, 311,* 47–52. https://doi.org/10.1126/science.1110600.
22. Lee, R., Wolpert, D. H., Bono, J., et al. (2013). Counter-factual reinforcement learning: How to model decision-makers that anticipate the future. *Decision making and imperfection* (pp. 101–128). Berlin Heidelberg: Springer.
23. Fujiwara-Greve, T., & Nielsen, C. (1999). Learning with forward looking players. *Keio Economic Society Discussion Paper*.
24. de Weerd, H., Verbrugge, R., & Verheij, B. (2014). Theory of mind in the Mod game: An agent-based model of strategic reasoning. *ECSI*.
25. Sato, Y., Akiyama, E., & Farmer, J. D. (2002). Chaos in learning a simple two-person game. *Proceedings of the National Academy of Sciences, 99,* 4748–4751.
26. Sato, Y., & Crutchfield, J. P. (2003). Coupled replicator equations for the dynamics of learning in multiagent systems. *Physical Review E, 67,* 015206.
27. Kleinberg, R., Ligett, K., Piliouras, G., & Tardos, É. (2011). Beyond the Nash equilibrium barrier. *Symposium on Innovations in Computer Science (ICS)*.
28. Galla, T., & Farmer, J. D. (2013). Complex dynamics in learning complicated games. *PNAS, 110,* 1232–1236. https://doi.org/10.1073/pnas.1109672110.
29. Frey, S., & Goldstone, R. L. (2013). Cyclic game dynamics driven by iterated reasoning. *PLoS ONE*. https://doi.org/10.1371/journal.pone.0056416.
30. Frith, C. D., & Frith, U. (2008). Implicit and explicit processes in social cognition. *Neuron, 60,* 503–510. https://doi.org/10.1016/j.neuron.2008.10.032.

31. Helbing, D., & Molnár, P. (1995). Social force model for pedestrian dynamics. *Physical Review E, 51,* 4282–4286. https://doi.org/10.1103/physreve.51.4282.
32. Goldstone, R. L., Wisdom, T. N., Roberts, M. E., & Frey, S. (2013). Learning along with others. *Psychology of learning and motivation* (pp. 1–45). Amsterdam: Elsevier.
33. Goldstone, R. L., Roberts, M., Mason, W., & Gureckis, T. (2010). Collective search in concrete and abstract spaces. In T. Kugler, J. C. Smith, T. Connolly, & Y. Son (Eds.), *Decision modeling and behavior in complex and uncertain environments* (pp. 277–308). New York: Springer.
34. Sumpter, D. J. T. (2010). *Collective animal behavior*. Princeton: Princeton University Press.
35. Couzin, I. D., & Krause, J. (2003). Self-organization and collective behavior in vertebrates. *Advances in the Study of Behavior, 32,* 1–75.
36. Nagel, R. (1995). Unraveling in guessing games: An experimental study. *American Economic Review, 85,* 1313–1326.
37. Ho, T.-H., Camerer, C. F., & Weigelt, K. (1998). Iterated dominance and iterated best response in experimental "p-beauty contests". *American Economic Review, 88,* 947–969.
38. Coricelli, G., & Nagel, R. (2009). Neural correlates of depth of strategic reasoning in medial prefrontal cortex. *Proceedings of the National Academy of Sciences, 106,* 9163.
39. Arad, A., & Rubinstein, A. (2012). The 11–20 money request game: A level-*k* reasoning study. *American Economic Review, 102,* 3561–3573. https://doi.org/10.1257/aer.102.7.3561.
40. Camerer, C. F., Ho, T.-H., & Chong, J.-K. (2002). A cognitive hierarchy theory of one-shot games. *UCLA Department of Economics Working Paper* (pp. 1–38).
41. Duffy, J., & Nagel, R. (1997). On the robustness of behaviour in experimental beauty contest games. *The Economic Journal, 107,* 1684–1700.
42. Frey, S., & Goldstone, R. L. (2011). Going with the group in a competitive game of iterated reasoning. *Cognitive Science Society, 33,* 6.
43. Robinson, D., & Goforth, D. (2005). *The topology of the 2 × 2 games*. London: Routledge.
44. Crawford, V. P., & Iriberri, N. (2007). Fatal attraction: Salience, naivete, and sophistication in experimental "Hide-and-Seek" games. *American Economic Review, 97,* 1731–1750.
45. Heap, S. H., Arjona, D. R., & Sugden, R. (2014). How portable is level-0 behavior? A test of level-*k* theory in games with non-neutral frames. *Econometrica, 82,* 1133–1151.
46. Crawford, V. P. (2014). A comment on "How portable is level-0 behavior? A test of level-*k* theory in games with non-neutral frames" by Heap, Rojo-Arjona, and Sugden. Unpublished Manuscript. Available at http://econweb.ucsd.edu/~vcrawfor/HeapComment.pdf.
47. Camerer, C. F., Ho, T.-H., & Chong, J. (2004). A cognitive hierarchy model of games. *Quarterly Journal of Economics, 119,* 861–898.
48. Friedman, J. W. (1971). A non-cooperative equilibrium for supergames. *The Review of Economic Studies, 38,* 1–12. https://doi.org/10.2307/2296617.
49. Rubinstein, A. (1979). Equilibrium in supergames with the overtaking criterion. *Journal of Economic Theory, 21,* 1–9. https://doi.org/10.1016/0022-0531(79)90002-4.
50. Aumann, R. J., & Shapley, L. S. (1994). Long-term competition—A game-theoretic analysis. *Essays in game theory* (pp. 1–15). New York, NY: Springer.
51. Fudenberg, D., & Maskin, E. (1986). The folk theorem in repeated games with discounting or with incomplete information. *Econometrica, 54,* 533. https://doi.org/10.2307/1911307.
52. Benoit, J.-P., & Krishna, V. (1985). Finitely repeated games. *Econometrica, 53,* 905. https://doi.org/10.2307/1912660.
53. Abreu, D., Dutta, P. K., & Smith, L. (1994). The folk theorem for repeated games: A NEU condition. *Econometrica, 62,* 939. https://doi.org/10.2307/2951739.
54. Denzau, A. T., & North, D. C. (1994). Shared mental models: Ideologies and institutions. *Kyklos, 47,* 3–31. https://doi.org/10.1111/j.1467-6435.1994.tb02246.x.
55. Richards, D. (2001). Coordination and shared mental models. *American Journal of Political Science, 45,* 259–276.
56. Nagel, R., Bühren, C., & Frank, B. (2017). Inspired and inspiring: Hervé Moulin and the discovery of the beauty contest game. *Mathematical Social Sciences, 90,* 191–207.
57. Palacios-Huerta, I., & Volij, O. (2009). Field centipedes. *American Economic Review, 99,* 1619–1635. https://doi.org/10.1257/aer.99.4.1619.
58. Stahl, D. O., & Wilson, P. W. (1994). Experimental evidence on players' models of other players. *Journal of Economic Behavior and Organization, 25,* 309–327.
59. Heinrich, T., & Wolff, I. (2012). Strategic reasoning in hide-and-seek games: A note. *Thurgau Research Paper Series*.

60. Agranov, M., Potamites, E., Schotter, A., & Tergiman, C. (2012). Beliefs and endogenous cognitive levels: An experimental study. *Games and Economic Behavior, 75,* 449–463. https://doi.org/10.1016/j.geb.2012.02.002.
61. Frey, S. (2013). Complex collective dynamics in human higher-level reasoning (Doctoral dissertation, pp. 1–324). Retrieved from *Proquest* (3599175).
62. Kocher, M., Sutter, M., & Wakolbinger, F. (2014). Social learning in beauty-contest games. *Southern Economic Journal, 80,* 586–613.
63. Alaoui, L., & Penta, A. (2016). Endogenous depth of reasoning. *Review of Economic Studies, 83,* 1297–1333. https://doi.org/10.1093/restud/rdv052.