



# Perceptual Learning, Cognition, and Expertise

Philip J. Kellman<sup>\*1</sup>, Christine M. Massey<sup>\*\*</sup>

<sup>\*</sup>Department of Psychology, University of California, Los Angeles, CA, USA

<sup>\*\*</sup>Institute for Research in Cognitive Science, University of Pennsylvania, Philadelphia, PA, USA

<sup>1</sup>Corresponding author: E-mail: Kellman@cognet.ucla.edu

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## Abstract

Recent research indicates that perceptual learning (PL)—experience-induced changes in the way perceivers extract information—plays a larger role in complex cognitive tasks, including abstract and symbolic domains, than has been understood in theory or implemented in instruction. Here, we describe the involvement of PL in complex cognitive tasks and why these connections, along with contemporary experimental and neuroscientific research in perception, challenge widely held accounts of the relationships among perception, cognition, and learning. We outline three revisions to common assumptions about these relations: 1) Perceptual mechanisms provide complex and *abstract* descriptions of reality; 2) Perceptual representations are often

*amodal*, not limited to modality-specific sensory features; and 3) Perception is *selective*. These three properties enable relations between perception and cognition that are both synergistic and dynamic, and they make possible PL processes that adapt information extraction to optimize task performance. While PL is pervasive in natural learning and in expertise, it has largely been neglected in formal instruction. We describe an emerging PL technology that has already produced dramatic learning gains in a variety of academic and professional learning contexts, including mathematics, science, aviation, and medical learning.



## 1. INTRODUCTION

On a good day, the best human chess grandmaster can defeat the world's best chess-playing computer. Not every time, but sometimes. The computer program is relentless; every second, it examines upward of 200 million possible moves. Its makers incorporate sophisticated methods for evaluating positions, and they implement strategies gotten from grandmaster consultants. Arrayed against these formidable techniques, it is surprising that any human can compete at all.

If, like the computer, humans played chess by searching through possible moves, pitting human versus computer would be pointless. Estimates of human search in chess suggest that even the best players examine on the order of four possible move sequences, each about four plies deep (where a ply is a pair of turns by the two sides). That estimate is *per turn*, not per second, and a single turn may take many seconds. If the computer were limited to 10 s of search per turn, its advantage over the human would be about 1,999,999,984 moves searched per turn.

Given this disparity, how can the human even compete? The accomplishment suggests information-processing abilities of remarkable power but mysterious nature. Whatever the human is doing, it is, at its best, roughly equivalent to 2 billion moves per second of raw search. It would not be overstating to describe such abilities as “magical.”

We have not yet said what abilities make this possible, but before doing so, we add another observation. Biological systems often display remarkable structures and capacities that have emerged as evolutionary adaptations to serve particular functions. Compared to flying machines that humans have invented, the capabilities of a dragonfly, hummingbird, or mosquito are astonishing. Yet, unlike anatomical and physiological adaptations for movement, the information-processing capabilities we are considering are all the more remarkable because it is unlikely that they evolved for one particular task. We did not evolve to play chess. What explains human attainments

in chess are highly general abilities that contribute to learned expertise in many domains. Such abilities may have evolved for ecologically important tasks, but they have such power and generality that humans can become remarkably good in almost any domain involving complex structure.

What abilities are these? They are abilities of perceptual learning (PL). The effects we are describing arise from experience-induced changes in the way perceivers pick up information. With practice in any domain, humans become attuned to the relevant features and structural relations that define important classifications, and over time, we come to extract these with increasing selectivity and fluency.

The existence of PL and its pervasive role in learning and expertise say something deeply important about the way human intelligence works. What it says violates common conceptions that view perception and learning as separate and nonoverlapping processes. It is common to think of perception as delivering basic information in a relatively unchanging way. According to this view, high-level learning happens elsewhere—in committing facts to memory, acquiring procedures, or generating more complex or abstract products from raw perceptual inputs by means of reasoning processes. Contemporary experimental and neuroscientific research in perception, as well as new discoveries in PL, require revision of these assumptions in at least three ways: 1) perceptual mechanisms provide complex and abstract descriptions of reality, overlapping and interacting deeply with what have traditionally been considered “higher” cognitive functions; 2) the representations generated by these perceptual mechanisms are not limited to low-level sensory features bound to separate sensory modalities; and 3) what perception delivers is not fixed, but progressively changing and adaptive.

We return to the first two ideas later on, but consider now what is implied by the third idea, the idea of PL. We can understand the adaptive nature of our perceptual abilities by way of contrast. Suppose we developed a set of algorithms in a computer vision system to recognize faces. The system would be structured to take input through a camera and perform certain computations on that input. If it worked properly, when we used the system for the thousandth time, it would carry out these computations in the same way as it did its first time. It is natural to think of a perceiving system as set up to acquire certain inputs and perform certain computations, ultimately delivering certain outputs.

Our brains do not work this way. If recognizing faces is the task, the brain will leverage ongoing experience to discover which features and patterns make a difference for important face classifications. Over time, the

system will become selectively attuned to extract this information and take it in in bigger chunks. (This is true even for perceptual abilities which, like face perception, likely have innate foundations.) With appropriate practice, this information extraction will become faster and more automatic. The automatization of basic information pickup paves the way for the discovery of even more complex relations and finer detail, which in turn becomes progressively easier to process (Bryan & Harter, 1899). This cyclic process can be a positive feedback loop: Improvements in information extraction lead to even more improvements in information extraction. The resulting abilities to see at a glance what is relevant, to discern complex patterns and finer details, and to do so with minimal cognitive load, are hallmarks of expertise in all domains where humans attain remarkable levels of performance.

It is likely that this type of learning comprises a much bigger part of the learning task in many domains than has been understood in theoretical discussions of learning or implemented in methods of instruction. What is being discovered about PL has implications for learning and instruction that parallel what researchers in artificial intelligence have discovered, “that, contrary to traditional assumptions, high-level reasoning requires very little computation, but low-level sensorimotor skills require enormous computational resources” ([http://en.wikipedia.org/wiki/Moravec's\\_paradox](http://en.wikipedia.org/wiki/Moravec's_paradox)). An artificial intelligence researcher, Hans Moravec, elaborated this idea in what has come to be known as “Moravec’s Paradox” (Moravec, 1988):

*Encoded in the large, highly evolved sensory and motor portions of the human brain is a billion years of experience about the nature of the world and how to survive in it. The deliberate process we call reasoning is, I believe, the thinnest veneer of human thought, effective only because it is supported by this much older and much more powerful, though usually unconscious, sensorimotor knowledge. We are all prodigious Olympians in perceptual and motor areas, so good that we make the difficult look easy. Abstract thought, though, is a new trick, perhaps less than 100 thousand years old. We have not yet mastered it. It is not all that intrinsically difficult; it just seems so when we do it.*

In what follows, we will add elaborations of two kinds to Moravec’s Paradox. First, our Olympian perceptual abilities are astounding because they give us access to a great many of the abstract relations that underlie thought and action. “Sensorimotor knowledge” does not convey the scope and power of what perceptual mechanisms deliver. Not only is explicit abstract thinking possibly a newer evolutionary acquisition, but the work of abstraction is not exclusively the province of thinking processes alone. Much of thinking turns out to be seeing, if seeing is properly understood.

The second elaboration is that the evolutionary heritage that makes us perceptual Olympians involves not only fixed routines, but perceptual systems that change—that attune, adapt, and discover to optimize learning, problem solving, and complex task performance. These changes comprise a much larger component of learning and expertise than is usually understood in learning research. Such an understanding of PL has been even more conspicuously missing from the efforts to improve school learning and other formal instructional efforts.

In this chapter, we describe recent work on PL, with a particular focus on its relation to complex cognitive tasks. One important goal is to describe how PL relates to perception, cognition, and learning. Some of the domains in which we apply PL, such as mathematics, will seem distant from perception to many readers. Thus, the theoretical underpinnings of the effort deserve to be spelled out, and doing so may facilitate the understanding of current efforts and continuing progress in these areas. Making the basic connections here is important because the emerging understanding of PL has broad implications throughout the cognitive and neural sciences. Both understanding PL, and using it to improve learning, depend on coherent accounts of the relation between perception, cognition, and learning. A second aim of this chapter, building on the first, is to describe an emerging technology of PL that has many applications and offers the potential to address missing dimensions of learning and accelerate the growth of expertise in many domains. A large and growing research literature suggests that PL effects are pervasive in perception and learning, and that they profoundly affect tasks from the pickup of minute sensory detail to the extraction of complex and abstract relations in complex cognitive tasks. PL thus furnishes a crucial basis of human expertise, from accomplishments as commonplace as skilled reading to those as rarified as expert medical diagnosis, mathematical expertise, grandmaster chess, and creative scientific insight.

The article is organized as follows: In the next section, we consider the information-processing changes that are produced by PL. These have most often been examined in tasks that involve either low-level sensory discriminations or real-world tasks that obviously depend on perceptual discrimination (e.g. detecting pathology in radiologic images). Using the example of PL in mathematics learning, Section 3 extends PL to higher level symbolic cognitive tasks, in which PL has seldom been considered. Understanding the role of PL in such tasks requires a revised account of the relations of perception, cognition, and learning. In Section 4, we argue that the common conceptions of these processes and their relations do not provide a

satisfactory foundation for understanding high-level PL effects, primarily because they are based on outdated ideas about perception. Drawing on more recent views, we describe a framework for understanding PL components of high-level cognitive tasks that is rooted in the amodal and abstract character of perception itself. With this framework in hand, we consider more fully in Section V the applications of PL to instruction.



## 2. PERCEPTUAL LEARNING EFFECTS

Perceptual learning refers to experience-induced improvements in the pickup of information (E. Gibson, 1969). The fundamental observation is that perceptual pickup is not a static process. After an intensive period of research in the 1960s and a somewhat dormant period for two decades afterward, PL has become an area of concentrated focus in the cognitive and neural sciences. The relative neglect and occasional focus on PL in the history of learning research and its recent emergence have been described elsewhere, as have issues of modeling PL and understanding its neural bases (for a review, see Kellman & Garrigan, 2009). Another important question has been the relation between simple laboratory tasks involving PL and more complex, real-world tasks typically involving the extraction of invariance amidst variation; recent work suggests that all of these tasks partake of a unified learning process in which the discovery of relevant information and its selective extraction are key notions (Ahissar, Laiwand, Kozminsky, & Hochstein, 1998; Garrigan & Kellman, 2008; Li, Levi, & Klein, 2004; Mollon & Danilova, 1996; Petrov, Doshier, & Lu, 2005; Zhang et al., 2010). In the present discussion, we build on this recent work but do not revisit it. Here, we focus on the range of effects produced by the PL, before turning to more general issues of how these relate to basic notions of perception, cognition, and learning.

A wealth of research now supports the notion that, with appropriate practice, the brain progressively configures information extraction in any domain to optimize task performance. What are the changes involved? The list involves a variety of distinguishable effects that serve to improve performance. Kellman (2002) argued that these effects fall into two categories: *discovery* and *fluency* effects. Discovery effects involve finding what information is relevant to a domain or classification. Fluency effects involve coming to extract information with greater ease, speed, or reduced cognitive load. Table 4.1 summarizes some of the changes between novices and experts that occur from PL. Discovery effects

**Table 4.1** Some Characteristics of Expert and Novice Information Extraction. Discovery effects involve learning and selectively extracting features or relations that are relevant to a task or classification. *Fluency* effects involve learning to extract relevant information faster and with lower attentional load (see text).

	Novice	Expert
<b><i>Discovery Effects</i></b>		
Selectivity	Attention to irrelevant and relevant information	Selective pickup of relevant information Filtering/inhibition of irrelevant information
Units	Simple features	Larger chunks Higher-order relations
<b><i>Fluency Effects</i></b>		
Search type:	Serial processing	Increased parallel processing
Attentional load:	High	Low
Speed:	Slow	Fast

include the fundamental idea of *selection* (Gibson, 1969; Petrov et al., 2005): We discover and pick up the information relevant to a task or classification, ignoring, or perhaps inhibiting (Kim, Imai, Sasaki, & Watanabe, 2012; Wang, Cavanagh & Green, 1994) available information that is irrelevant. We come to extract information in larger chunks, forming and processing higher-level units (Chase & Simon, 1973; Goldstone, 2000). Most profoundly (and mysteriously), we come to discover new and often complex relationships in the available information to which we were initially insensitive (Chase & Simon, 1973; Kellman, 2002). These discovery processes are pervasive in early learning. When a child learns what a dog, toy, or truck is, this kind of learning is at work. From a number of instances, the child extracts relevant features and relations. These allow later recognition of previously seen instances, but more important, even a very young child quickly becomes able to categorize *new* instances. Such success implies that the learner has discovered the relevant characteristics or relations that determine the classification. As each new instance will differ from previous ones, learning also includes the ignoring of irrelevant differences.

*Fluency effects* refer to changes in the efficiency of information extraction. Practice in classifying leads to fluent and ultimately automatic processing (Schneider & Shiffrin, 1977), where automaticity in PL is defined as the

ability to pick up information with little or no sensitivity to attentional load. As a consequence, perceptual expertise may lead to more parallel processing and faster pickup of information.

The distinction between discovery and fluency effects is not always perfectly clear. For example, becoming selective in the use of information (a discovery effect) increases efficiency and improves speed (fluency effects). It does seem, however, that there are clear cases of each category. In one of the earliest relevant studies, [Bryan and Harter \(1899\)](#) reported that telegraph operators learning to receive Morse code reached plateaus in performance, but that continuing practice while at a plateau appeared to pave the way for substantial new gains in performance. Their interpretation is that the eventual improvements in performance came from automaticity—operators coming to extract the same information with less cognitive load, ultimately enabling them to discover more complex relations in the input. This interpretation is consistent with a relatively pure fluency improvement, that is, with practice at a certain point not changing the information being extracted, but allowing its extraction with reduced attentional load ([Schneider & Shiffrin, 1977](#)). The continuing cycle of discovery and fluency described by Bryan and Harter—discovery leading to improved performance, followed by improved fluency, leading in turn to higher level discovery—may be the driver of remarkable attainments of human expertise in many complex tasks.



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### **3. PERCEPTUAL LEARNING IN MATHEMATICS: AN EXAMPLE**

There is a common view about the relation of perception and cognition. In a hierarchy of cognitive processes, perception is typically considered “low-level,” where “higher” cognitive processes encompass categorization, thinking, reasoning, etc. Eleanor Gibson, who pioneered the field of PL, thought of it as a pervasive contributor to expertise, giving examples as varied as chick sexing, wine tasting, map reading, X-ray interpretation, sonar interpretation, and landing an aircraft. Even these examples, however, are mostly confined to tasks where the major task component is classifying perceptual inputs based on subtle kinds of information. For most of these examples, one might still maintain a notion of perception as handing off results of basic feature detection, which then become the raw material for conceptual analysis, cognitive inferences, and high-level thinking.



Recent work, however, indicates that PL is strongly involved even in very high-level cognitive domains, such as the learning and understanding of mathematics (Kellman, Massey, & Son, 2009; Landy & Goldstone, 2007). Learning in these domains involves a variety of cognitive processes, but attaining expertise depends substantially on pattern recognition and fluent processing of structure, as well as mapping across transformations (e.g. in algebra) and across multiple representations (e.g. graphs and equations). In fact, given conventional instruction, the PL components of expertise may be disproportionately responsible for students' difficulties in learning (Kellman et al., 2009). Although this research area is relatively new, findings indicate that even short PL interventions can accelerate the fluent use of structure, in contexts such as the mapping between graphs and equations (Kellman et al., 2008; Silva & Kellman, 1999), apprehending molecular structure in chemistry (Wise, Kubose, Chang, Russell, & Kellman, 2000), processing algebraic transformations, and understanding fractions and proportional reasoning (Kellman et al., 2009).

The structures and relations that are relevant to PL in these domains are more abstract and complex than what we normally think of as being processed perceptually. As an example, Kellman et al. (2009) studied algebra learning using a perceptual learning module (PLM) designed to address the seeing of structure in algebra. Participants were 8th and 9th graders at midyear in Algebra I courses. Students at this point in their learning show a characteristic pattern. Given simple equations to solve, such as  $x + 4 = 12$ , accuracy is high, with an average across participants of around 80% correct solutions. Remarkably, however, students at this stage take an average of about 28 s per problem! This pattern suggests that conventional instruction does a good job of addressing the declarative and procedural aspects of solving algebraic equations. Students know they should “get x alone on one side,” and “do the same operation to both sides of the equation,” and they were able to accomplish these goals with high accuracy. Their response times, however, suggest that we may underestimate the seeing problem in algebra learning. Someone with much greater experience looks at  $x + 4 = 12$  and sees the answer at a glance. This kind of ability can reach higher and higher levels, supporting greater expertise, as illustrated in this example:

$$\mu = \frac{(4\phi - 2\phi\psi)}{(2 - \psi)(2\phi)}.$$

Given that this is a single equation with two unknowns, one might think at first glance that the problem does not permit a numerical solution for  $\mu$ , but a more practiced observer may easily see that the equation permits easy simplification, and  $\mu = 1$ . In this case, even the relative unfamiliarity of the symbols used may make the seeing problem harder. Without changing anything mathematical, compare

$$m = \frac{(4x - 2xy)}{(2 - \gamma)(2x)}.$$

If this equation still has you reaching for pen and paper, seeing the structure may be better illustrated in this simpler version:

$$m = \frac{(x - x\gamma)}{(x)(1 - \gamma)}.$$

These examples all involve the distributive property of multiplication over addition. However, being able to enunciate this property would not produce fluent recognition of the distributive structure. Conceptually, and even computationally, these examples are all very similar, but you may have noticed the relevant structure more easily in one case than another. Improved encoding of relevant structure and potential transformations in equations is a likely result of PL, one that is difficult to address in conventional instruction.

Following this kind of intuition, we developed our *Algebraic Transformations PLM* in order to apply PL methods to improve students' pattern processing and fluency in algebra. We developed a classification task in which participants viewed a target equation or expression and made speeded judgments about which one of a set of possible choices represented an equivalent equation or expression, produced by a valid algebraic transformation. A key goal of this PLM was to contrast the declarative knowledge components (facts and concepts that can be verbalized) with the idea of "seeing" in algebra. The goal was to get students to see the structure of expressions and equations, and relations among them, in order to use transformations fluently.

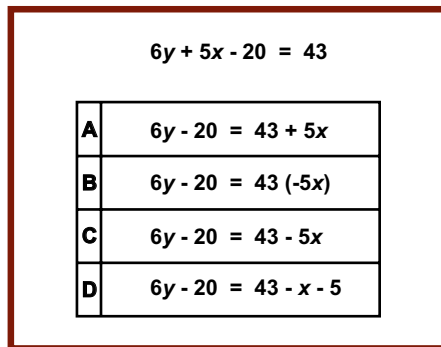
In the *Algebraic Transformations PLM*, we did not ask students to solve problems. Instead, we devised a classification task that exercised the extraction of structure and the seeing of transformations. On each trial, an equation appeared, and the student had to choose which one of several options below was a legal transformation. An example is shown in [Figure 4.1](#). In addition to testing whether practice in the PLM improved accuracy and

fluency in recognizing transformations, we also examined whether students would be able to transfer learning to solving algebraic equations.

This study was carried out with forty-two 8th and 9th grade students at midyear in an Algebra 1 course. Students participated in two 40-min learning sessions using the *Algebraic Transformations PLM*. On each trial, they were shown a target equation and were asked to select which of four choices could be correctly derived by performing a legal algebraic transformation on the target. Students were given feedback after each trial indicating whether or not they had chosen the correct answer. Incorrect answers were followed by an interactive feedback screen in which students' attention was focused on the relevant transformation.

The task that formed the core of the PLM—matching an equation to a valid transformation—is directly useful to development of pattern recognition and skill in algebra. The PLM produced dramatic gains for virtually all students on this task, with accuracy changing from about 57% on initial learning trials to about 85% at the end of PLM usage, and response times per problem reduced by about 55%, from nearly 12 s per problem to about 7 s, suggesting the development of fluency in processing symbolic structure of equations.

Perhaps more remarkable was the transfer to actual algebra problem solving. Although students did not receive any practice in solving equations during the learning phase, the relatively brief intervention aimed at seeing transformations produced a dramatic reduction in the post-test equation solving time—from about 28 s per problem to about 12.5 s per problem (Figure 4.2, right panel). A delayed post-test showed that these gains were lasting: The average solving time was actually slightly faster than in the immediate post-test when tested after a 2-week interval. There was



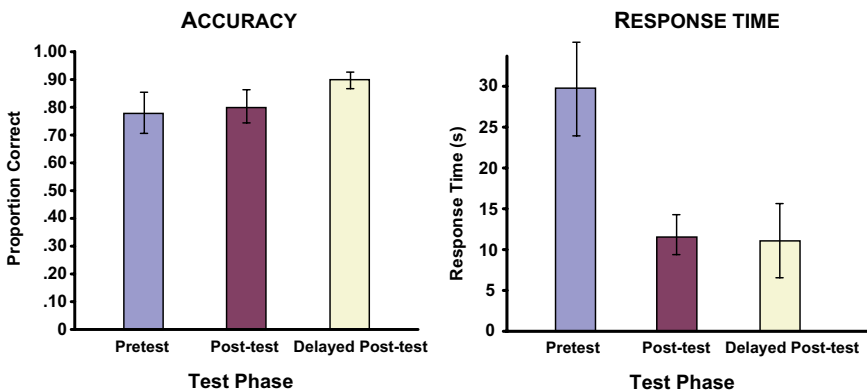
$6y + 5x - 20 = 43$	
<b>A</b>	$6y - 20 = 43 + 5x$
<b>B</b>	$6y - 20 = 43 (-5x)$
<b>C</b>	$6y - 20 = 43 - 5x$
<b>D</b>	$6y - 20 = 43 - x - 5$

**Figure 4.1** Example of a Problem Display in the Algebraic Transformations PLM (see text).

also some indication that accuracy in equation solving, already high at pretest, received some benefit in the delayed post-test (Figure 4.2, left panel).

The idea that mathematical understanding has an important PL component may seem counterintuitive, for many reasons. If perception is about properties such as brightness, color, the orientation of edges, or even the locations of objects and surfaces, how is this relevant to a mathematics class? These perceptual contents might at best serve up the occasional concrete example, but they hardly encompass mathematical ideas. On traditional views, most of mathematical thinking, and the instructional methods used to teach it, involve declarative knowledge and procedures. Perception may serve the banal role of allowing the student to see the markings on the chalkboard, but the processing of mathematical ideas must surely be farther up in the cognitive hierarchy! There would seem to be a gap between the basic and concrete information furnished by the senses and the abstract conceptual content of mathematics. The simple difference between the level or types of information that perception is presumed to furnish and what is required for abstract thinking seems a formidable obstacle to the kind of connection we are making here.

But it is not the only obstacle. Mathematics has inherently symbolic aspects. The symbols in an equation have an *arbitrary* relation to the ideas they represent. Unlike the functional properties of objects and events in the world, the meanings of mathematical ideas would seem remote from



**Figure 4.2 Results of Algebraic Transformations PLM Study for the Transfer Task of Solving Algebraic Equations.** Data for pretest, post-test, and delayed post-test are shown for accuracy (left panel) and response time (right panel). Error bars indicate  $\pm 1$  standard error of the mean (Adapted from Kellman, Massey & Son, *TopiCS in Cognitive Science*, 2009; *Cognitive Science Society, Inc.*, p. 14). For a color version of this figure, the reader is referred to the online version of this book.

stimulus information reaching perceptual systems. Moreover, much of the expertise conferred by PL may be implicit (e.g. try describing to a stranger how you recognize your sister's voice on the telephone), whereas mathematics is in many respects an extremely explicit discipline. Steps must be justified and proofs must be offered. Even assuming the relevance of PL to complex tasks, one might still wonder about the application to symbolic, explicit domains such as mathematics.

Many of these objections have straightforward answers. Even if they involve symbolic content, mathematical representations pose important information extraction requirements and challenges. Characteristic difficulties in mathematics learning may directly involve issues of discovery and fluency aspects of PL. A number of studies indicate the role of PL in complex cognitive domains, such as mathematics (Kellman et al., 2009; Landy & Goldstone, 2007; Silva & Kellman, 1999), language or language-like domains (Gomez & Gerken, 1999; Reber, 1993; Reber & Allen, 1978; Saffran, Aslin, & Newport, 1996), chess (Chase & Simon, 1973), and reading (Baron, 1978; Reicher, 1969; Wheeler, 1970). Some have asserted that in general, abstract concepts have crucial perceptual foundations (Barsalou, 1999; Goldstone, Landy, & Son, 2008; Prinz, 2004).

The extensive use of tangible representations in mathematics, science, and other abstract conceptual domains is also a bit of a giveaway. Hardly two steps into considering a complicated problem in mathematics, science, economics, or other quantitative disciplines we construct a graph or a diagram, if not several. One's facility in dealing with these representations obviously changes with experience, in obscure ways that go beyond being able to explain the basics of how the diagram represents information. We seem to grapple with complex ideas in mathematics and science by using spatial, configural, and sometimes temporal structures (i.e. simulations) that draw on representational capacities rooted in our perceptions of spatial and temporal structure in the world. A graph of the change of world temperature over time is a spatial object, and the patterns therein are comprehended by grasping spatial relations, although neither temperature nor time is a spatial notion. Reliably accompanying the use of these structures and representations are powerful, general capacities to learn to detect relations and become able to fluently select information that is important within a domain: Perceptual learning.

Still, we are stuck with the first objection. Perception, as commonly understood, just seems to be at the wrong level for explaining comprehension in mathematics. Maybe the connection is intended as some of kind of metaphor. If one conceives of perception as consisting of separate sense

modalities, then what we obtain through vision must somehow be built from sensory experiences of brightness and color. In audition, we are presumably extracting sequences and combinations of loudness and pitch. In algebra class, one should listen to the teacher's voice and look at the blackboard, but surely algebra is not about arrays of color, brightness, loudness, or pitch.

Later in this chapter, we will have more to say about PL technology and the potential for radically improving learning by integrating methods that accelerate PL with conventional instruction. For now, however, we focus on what appears to be most perplexing in our example of PL in complex cognitive domains. If it is surprising that changing the perceiver can be the key to advancement in domains such as mathematics, it is because there is work to do in clarifying the relation of perception to learning and cognition. This is the focus of the next section.



#### **4. PERCEPTION, COGNITION, AND LEARNING**

Continuing scientific progress and practical applications of PL will be facilitated by a better understanding of the relations between perception, cognition, and learning. One might assume that these relations are well understood, but in fact they are not. A primary reason is that progress in understanding perception in the past several decades necessitates a rethinking of some of these relations, invalidating some ways of thinking and paving the way for new insights.

As we mentioned above, commonly held views of perception would suggest that the products of perceiving are too low level to have consequences for abstract thinking and learning. Thus, before the last few years, if someone suggested a role for perception in learning mathematics, it would involve using shaded diagrams to illustrate fraction concepts or manipulatives that might allow learners to have some concrete realization of adding numbers. These applications are quite different from the idea of a general learning mechanism by which learners progressively change the way they extract structure and relations from symbolic equations, or gain competency in mapping structure across differing mathematical representations, or come to selectively attend to important relations, rather than irrelevancies, in a measurement problem.

In recent years, there have been trends in cognitive science arguing for a close relation between perception and cognition. This work includes empirical findings that implicate perceptual structure as being involved in processing abstract ideas (Landy & Goldstone, 2007) and other research indicating modal sensory activations accompanying cognitive tasks such as

sentence verification (van Dantzig, Pecher, Zeelenberg, & Barsalou, 2008). There have been accompanying theoretical proposals that suggest that high-level cognition depends fundamentally on perception, including the ideas of perceptual symbol systems (PSS; Barsalou, 1999) and the notion of embodied cognition. We believe that these accounts share important elements, and all are an improvement on an earlier, implicit general view of perception being detached from thinking.

In our view, however, none of these efforts provides a suitable basis for understanding the relation of perception and PL to the rest of cognition and to complex learning domains. As a result, the situation is confusing. We have found this to be especially troubling in terms of connecting emerging findings in PL and PL technology in instruction with conventional ideas of cognition, teaching, and learning. The reason is that neither the older assumptions about how these relate nor most recent proposals in cognitive psychology provide a coherent basis for understanding the relation of PL to cognition in general. We briefly discuss some of these views and their problems before describing a more coherent, as well as simpler, account, one grounded in a contemporary understanding of perception.

#### 4.1. The Classical View of Perception

In classical empiricist theories of perception and perceptual development, widely shared for several centuries by many philosophers and psychologists, all meaningful perception (e.g. perception of objects, motion, and spatial arrangement) was held to arise from initially meaningless sensations. Meaningful perception was thought to derive from associations among sensations (e.g. Berkeley, 1709/1910; Locke, 1690/1971; Titchener, 1902) and with action (Piaget, 1952). In this view, all of perception is essentially a cognitive act, constructing meaning by associating sensations and connecting them with previously remembered sensations. A modern version of this view, widely shared in cognitive psychology, is satirized in a famous information-processing diagram in Ulric Neisser's book *Cognition and Reality* (Neisser, 1976), in which an input labeled "retinal image" is connected by arrows to successive boxes labeled "processing," "more processing," and "still more processing."

This view of perception came with its own view of PL. Essentially, on this view, *all* meaningful perception is a product of learning. Inferring the motion of an object from sensations encoded at different positions and times, or understanding the three-dimensional shape of an object by retrieving previously stored images gotten from different vantage points involve meaningless sensations combined with associative learning processes (e.g.

Locke, 1690/1971), or unconscious inference processes working on current and previously stored sensations (Helmholtz, 1864/1972).

## 4.2. Perceptual Symbol Systems

There have been clear trends among cognitive researchers to connect perception more closely to other cognitive processes or to uncover perceptual influences in cognitive tasks. Particularly influential has been the work of Barsalou on “perceptual symbol systems” (PSS). PSS comprise proposals to account for a number of important phenomena, including well-known difficulties of specifying formal, context-free criteria of inclusion in conceptual categories (e.g. what makes something a cat); the apparently dynamic, variable aspects of representations; and the engagement of cortical areas involved with perception during cognitive tasks.

According to PSS, the idea of nonperceptual, abstract thought does not really exist. Even our most abstract ideas are attained by reference to stored perceptual encodings. As Barsalou (1999) explains,

*...abstract concepts are perceptual, being grounded in temporally extended simulations of external and internal events. (Barsalou, 1999, p. 603)*

Specifically, when we think of an apparently abstract idea, that processing consists of running a “simulation,” which consists of “re-enactment in modality-specific systems”:

*The basic idea behind this mechanism is that association areas in the brain capture modality-specific states during perception and action, and then reinstate them later to represent knowledge. When a physical entity or event is perceived, it activates feature detectors in the relevant modality-specific areas. During visual processing of a car, for example, populations of neurons fire for edges, vertices and planar surfaces, whereas others fire for orientation, colour and movement. The total pattern of activation over this hierarchically organized distributed system represents the entity in vision (e.g. Zeki 1993; Palmer, 1999). Similar distributions of activation on other modalities represent how the entity feels and sounds, and the actions performed on it. (Barsalou, 2003, p. 1179)*

Barsalou contrasts this view with what he sees as the more typical view in cognitive science that information gotten through perception is “transduced” into amodal representations, where

*... an amodal symbol system transduces a subset of a perceptual state into a completely new representation language that is inherently nonperceptual. (Barsalou, 1999, p. 577)*

We believe that Barsalou and others have identified a key problem—a perceived disconnect between information processing involving most



cognitive processes and perception. The general idea that these are more closely coupled than often believed is consistent with considerable evidence and has opened up some important issues in these fields.

### 4.3. Problems for Understanding Perceptual Learning

Regrettably, both the classical view and more recent proposals about the relation between perception and cognition make poor foundations for understanding current approaches to and results of PL. Mathematics seems much more abstract than perception. Consider the applications of PL to mathematics that we described above. On the classical view, it is hard to relate the abstract structures in mathematics to the aggregates of sensations that are the harvest from perceiving. Mathematics seems to be the province of higher-level reasoning, not perception.

The situation is somewhat reversed from Barsalou's PSS view. Here, it is claimed that abstract ideas do not really exist off by themselves; what we think of as abstract thought really consists of activations of modality-specific features. On this account, all mathematics would be inherently perceptual. It is hard to see how it would be abstract, however. If the input contents are all modality specific, what is mathematics? Is mathematics visual? Is it auditory? Tactile? Mathematics does not really seem to be any of those things. From the PSS account, it could be argued that thinking about a mathematical idea involves running certain re-enactments of particular perceptual experiences. These are likely multimodal; they could have inputs from different modalities such as the sound of your teacher's voice in algebra class or the chalkmarks on the blackboard, or the feel of your pencil in your hand. Thinking about particular ideas in particular contexts would involve re-enacting (simulating) different subsets of stored perceptual records.

Both this approach and the classical approach have massive problems with *abstraction* and *selection*. Consider a student who is mastering the concept of slope in a PLM involving graphs, equations, and word problems. The student's task is to map a problem represented in one format, such as a graph, to the same mathematical structure as it is expressed in either an equation or a word problem. The student learns to extract a general idea that applies to new contexts, as well as structural invariants specific to representational types (Kellman, Massey & Son, 2009). In a graphic representation, the understanding of slope emerges as involving spatial directions: A positively sloped function increases in height from the left to the right; steeper increases show larger slopes, and so on. From mapping word problems onto graphs, the deeper understanding emerges that a positive slope

involves increases in one quantity as another quantity increases. As water is heated, a rising temperature over time implies a positive slope. One could well recall an experience of boiling water when one thinks about slope, but that would not help without some mechanism of specifying which parts of that experience constitute slope. The slope concept can apply to boiling water but is not about boiling water. It has been argued that the PSS framework involves insurmountable problems in that rerunning various perceptual records provides no mechanism for selecting a particular idea (Landau, 1999; Ohlsson, 1999). The problem is especially severe when the idea is an abstract one, such as slope. Meaningful learning here would involve a student being able to apply slope to novel situations (e.g. knowing what it would mean if there were a negative slope relating number of business startups to interest rates). It is hard to fathom how this understanding of a novel case could come from rerunning subsets of prior modality-specific activations. As Ohlsson (1999) puts it,

*A closely related difficulty for Barsalou's theory is that the instances of some concepts do not share any perceptible features. Consider furniture, tools, and energy sources. No perceptible feature recurs across all instances of either of these categories. Hence, those concepts cannot be represented by combining parts of past percepts. (Ohlsson, 1999, p. 630)*

PL in complex cognitive domains leads to selective extraction and fluent processing of abstract relations, such as slope. From transactions with individual cases, learners come to zero in on the properties, including abstract relations, that underlie important classifications. The process is PL, as it changes the way information is extracted. The learning is highly selective; selection is so fundamental to PL that Gibson (1969) used “differentiation learning” as a synonym for PL. Finally, the properties learned are abstract. Whether in chess, speech recognition, chemistry, or mathematics, PL often leads to selective, fluent extraction of relational and abstract information (Kellman & Garrigan, 2009).

Traditional views of perception and recent proposals regarding perception and cognition, such as PSS, do not appear to offer reasonable ways of understanding these aspects of PL. How can we understand them? To begin with, the answer can be found in a better understanding of perception itself.

#### **4.4. The Amodal, Abstract Character of Perception**

Both the classical view of perception and recent attempts to connect perception and cognition are hampered by a failure to understand the amodal, abstract character of perception.

Research and theory in perception over the past several decades have made it clear that perceptual systems are sensitive to complex relations in stimulation as their inputs, and they produce meaningful descriptions of objects, spatial layouts, and events occurring in the world (J. Gibson, 1966, 1979; Johansson, 1970; Marr, 1982; Scholl & Tremoulet, 2000). Most perceptual mechanisms develop from innate foundations or maturational programs and do not rely on associative learning to provide meaningful perception of structure and events (for a review, see Kellman & Arterberry, 1998). Many structural concepts that might earlier have been considered exclusively cognitive constructs have been shown to be rooted in perceptual mechanisms. Some of these include causality (Michotte, 1963), animacy (Johansson, 1973), and social intention (Heider & Simmel, 1944; Runeson & Frykholm, 1981; Scholl & Tremoulet, 2000).

These features of perception are difficult to reconcile with a shared assumption of classical views, PSS, and some other approaches that the products of perceiving are sets of sensory activations that are modality specific—that is, unique to particular senses. In PSS, for example, the definition of perceptual symbols requires that they be modality specific, consisting of records of “feature activations” (Barsalou, 1999, 2003). This approach to representation, according to Barsalou, replaces the amodal symbols common in other cognitive modeling, resulting in the view that there may be no truly abstract, amodal symbols at all.

Any approach of this sort is difficult to reconcile with the fact that most of the perceptual representations that are central to our thought and action have a distinctly nonsensory character. For example, as the Gestalt psychologists pointed out almost 100 years ago, the perceived shape of an object is something quite different from the collection of sensory elements it activates (Koffka, 1935). The problems with obtaining the products of perception from aggregates of sensory activations are well known (J. Gibson, 1979; Koffka, 1935; Kellman & Arterberry, 1998; Landau, 1999; Marr, 1982; Nanay, 2010; Ohlsson, 1999).

The solution of how to connect perception with abstract thought is not that abstract thought consists of simulations of sensory feature activations but that perception *itself* is amodal and abstract. The terms “modal” and “amodal” were in fact introduced in perception research by Michotte, Thines, and Crabbe (1964) with regard to these issues. Michotte et al. use *both modal* and *amodal* to refer to perceptual phenomena. In his classic work on visual completion, modal completion refers to cases in which the visual system fills in information that includes sensory properties, such

as brightness and color, whereas amodal completion refers to filling-in in which the object structure is represented perceptually but there is an absence of sensory properties. (The latter happens, for example, when an object is seen as continuing behind a nearer occluding object.) Michotte's view, supported by extensive research, is that both kinds of filling-in are accomplished by perceptual mechanisms, not processes of reasoning or cognition (Kanizsa, 1979; Keane, Lu, Papatomas, Silverstein, & Kellman, 2012; Michotte et al., 1964). In fact, both kinds of filling-in appear to depend on the same perceptual mechanisms (Kellman & Shipley, 1991; Kellman, Garrigan, & Shipley, 2005; Murray, Foxe, Javitt, & Foxe, 2004). In general, visual perception of ordinary surfaces and objects results in representations of complete objects and continuous surfaces, even when many parts of these are not represented in local sensory input due to occlusion or camouflage (Kanizsa, 1979; Kellman & Shipley, 1991; Michotte et al., 1964; Palmer, Kellman, & Shipley, 2006).

The issue here may be in part terminological. Barsalou (1999, 2003) defines perceptual symbols in general as necessarily "modal," and contrasts these with the nonperceptual or "amodal" symbols. His explication of modal perceptual symbols includes being the property of a single sense and being "analogical," in that such symbols are "represented in the same systems as the perceptual states that produced them. The structure of a perceptual symbol corresponds, at least somewhat, to the perceptual state that produced it" (Barsalou, 1999). One could explore the idea that Barsalou may be giving the terms "modal" and "amodal" new meanings and therefore there is no conflict with Michotte's ideas. On this view, anything vision does is "modal" because vision is one sense, as distinguished, for example, from audition. The nonsensory phenomena of visual object and surface perception, and so on, would simply be modal under these new definitions.

The different use of terms is accompanied by a difference in concept, however. The problem is clear in the proposals that perceiving an object consists of feature activations, such as neurons for edges, vertices, orientation, color, etc., and that "The total pattern of activation over this hierarchically organized distributed system represents the entity in vision." Barsalou's view is in many ways remarkably close to classical views of sensation and perception, as he notes (Barsalou, 1999, p. 578).

In the field of perception, Michotte's ideas were incorporated into the more comprehensive ecological, information-based theories of J. J. Gibson (1966, 1979). Gibson made the case that perceptual mechanisms have evolved to be sensitive, not to simple, local stimuli, but to higher order relations

(invariants) in stimulation that correspond to important environmental properties or important events involving the perceiver and the environment. Much of the important information is not even present in a particular, momentary sensory array (image). For example, variables in optic flow—the continuously transforming projection of the environment onto the eyes—specify the direction of travel of a moving observer, as well as the layout of surfaces ahead (Gibson, 1979; Warren & Hannon, 1988). In general, Gibson embraced the idea of perception, at least its most functionally important aspects, as “sensationless.”

An example of the extraction of complex relations by perceptual mechanisms to produce descriptions of high-level, abstract properties may help to make this idea intuitive. Johansson (1973) placed small lights (“point lights”) on the joints of a person, and filmed the person walking in the dark. When viewed by a human observer, there is a compelling and automatic percept of a person walking. Such displays may also convey information about gender or specific individuals. Many more complex events involving so-called *biological motion* have been shown to be quickly and effortlessly perceived, including dancing and jumping.

Any static view of the dots used in these displays conveys only a meaningless jumble. Moreover, dot displays, in momentary images or in motion, do not at all resemble any stored images (or sets of feature activations) we may have of actual walking (or dancing) persons. All the basic sensory features in these displays are, upon first presentation, brand new. Moreover, the observer represents perceptually a walking person and encodes in a durable fashion almost nothing about positions of particular dots in momentary images, or dot trajectories, that comprised the animation sequence. The fact that observers uniformly and automatically perceive meaningful persons and events in these displays indicates that our normal encoding of persons and events in the environment includes abstract relations of high complexity.<sup>1</sup> All these observations illustrate crucial and general aspects of perception: We do register sensory elements (and feature activations), but we do so as part of processes that extract complex and abstract relations relevant to detecting ecologically important properties of objects and events. It is these properties that are encoded; the basic sensory features are transient, quickly discarded, and, apart from the relations in which they participate, quite irrelevant to perception. These ideas that perceptual systems utilize complex relational

<sup>1</sup> They are complex enough that scientists who study computational vision have not yet been able to produce algorithms to approximate human performance in perceiving structure from point-light displays.

information as inputs and produce abstract, amodal representations as outputs are shared by virtually all contemporary ecological and computational work in perception (Hochberg, 1968; Kellman & Arterberry, 1998; Marr, 1982; Shepard, 1984; Pizlo, 2010) and are not subjects of serious dispute.

We should note specifically that this view of the outputs of perception as amodal, meaningful abstractions applies even to seemingly simple cases of perception. The idea that we could represent some object in the world, say, a car, in terms of sets of feature detectors activated in sensory areas, constitutes a vast and misleading simplification. It is true that early cortical areas in the visual system contain orientation-sensitive units that respond to retinally local areas of oriented contrast. So it may seem straightforward to assume that activations of such cells could represent the oriented edges of a car that we see. But this is a misunderstanding. The perceived orientation of an edge of a car in the world is actually the result of complex computations accomplished by perceptual mechanisms; it is not a readout of the outputs of early orientation-sensitive units. One reason is that capturing information about an edge in the world requires utilizing relations among many different orientation-sensitive units of different local orientations and scales (e.g. Lamme & Roelfsema, 2000; Sanada & Ohzawa, 2006; Wurtz & Lourens, 2000). Another problem is that the early neural units in vision encode two-dimensional orientations on the retina, not the three-dimensional orientations in space needed in our perceptual representations (for discussion, see Kellman et al., 2005). The most general version of the problem here, however, is that the word “orientation” means different things for the “feature detectors” of the basic vision scientist and the object “features” needed in cognitive models. The former are invariably *retinal*, meaning that the orientation-sensitive units in V1 that get activated depend on the orientation and position of contrast on the retina of the eye. This position and orientation information typically changes several times a second,<sup>2</sup> as it depends crucially on the position of the eyes in the head, the head on the body, and the body in the world. Thus, the correspondence between the orientation of an edge in the world and which orientation-sensitive units are firing in the brain is haphazard. Complex relations in the activities of orientation-sensitive units allow us to encode properties of objects in a world-centered coordinate system, but there is no reason to believe that we encode into any

<sup>2</sup> Even identifying an early cortical unit with a single retinal orientation is an oversimplification. In fact, early cortical units in vision have complex response profiles that include changes in their orientation sensitivity over periods <100ms, and they are sensitive to many other influences of context (Lamme & Roelfsema, 2000; Ringach, Hawken, & Shapley, 2003).

lasting form of storage any sensory records of the momentary activations of neurons in early cortical areas. In fact, there are reasons to believe that early activations of “feature detectors” in early visual processing are not accessible to even momentary conscious awareness (e.g. Crick & Koch, 1995; He & MacLeod, 2001) and cannot be accessed by learning mechanisms (Garrigan & Kellman, 2008). In short, the perception of a simple environmental property, such as the edge of a car, is a complex abstraction, based on relational information; the relations of this abstraction to the outputs of populations of detectors, such as the orientations signaled by neuron in early visual areas, are highly variable; and the latter are not preserved in any accessible outputs of the process. Elementary activations in sensory areas are not the elements of perceptual representations—not even the seemingly simple ones, such as orientation or color (see Zeki, Aglioti, McKeefry, and Berlucchi (1999) for a parallel argument regarding color). We would go so far as to say that the term “feature detector” has proven to be an unfortunate choice in sensory neuroscience. When construed to mean that early neural units signal the features of objects, surfaces or events in the world, it is a misunderstanding.

The transition from a view of perceptual representations as some kind of energy imprint on the sensory surfaces to a view of these representations as amodal and abstract parallels the developments in other sciences. Commenting on the ways in which quantum mechanics had changed conceptions of matter from continuous and concrete to something much more abstract, the philosopher Bertrand Russell put it: “It has begun to seem that matter, like the Cheshire Cat, is becoming gradually diaphanous and nothing is left but the grin, caused, presumably, by amusement at those who still think it is there.” The lingering view of perception, and the representations gotten from perception, as being fundamentally about local sensory activations is just like this. In contemporary views, sensory activations provide a medium from which perceptual mechanisms extract informative relations in order to represent in abstract fashion ecologically important structures of objects and events. The sensory Cheshire cat has proved similarly diaphanous, leaving nothing but the grin. (Even the grin, if we recall it from having seen a picture earlier, is an abstract structure, not an array of sensations).

#### **4.5. The Selective Character of Perception**

Earlier, we noted that selection is a key principle in PL. It is a crucial characteristic, because organisms are surrounded at any moment by a wealth of stimulation. The tasks they need to perform require highly selected subsets of this information, and sometimes require discovery of complex, subtle,

and abstract properties and relations. Moreover, we have limited immediate processing capacities, such that cognitive load is a major constraint on performance in most tasks, and conspicuously so in learning (Paas & van Merriënboer, 1994; Schneider & Shiffrin, 1977; Sweller & Chandler, 1991). Selective apprehension of information and improvements in fluency (speed, chunking, and automaticity or reduced load) with practice are both primary mechanisms by which humans cope with these limitations and major determinants of expertise in most domains.

As we noted above, information selection in classical views would be hard to accomplish because pickup was based on sensations, not information. Both fashioning abstract ideas out of associations of sensations and altering the information extraction process with experience are hard to fathom from this starting point. The situation is different but equally problematic from the PSS perspective. Again, if records of feature activations for whole episodes are what is picked up and what is stored, selection and isolation of invariants or distinguishing features pose an unsolved problem.

Fortunately, the problem is much more easily handled in contemporary views of perception, in which selection plays an important and intrinsic role. As we have seen, selective computation of abstract properties, from simple ones such as edge orientation in space, to more complex ones such as shape or sets of motion relationships that specify objects, surfaces or events, is a fundamental characteristic of perception (J. Gibson, 1979) and appears to be presumed by learning mechanisms as well (Garrigan & Kellman, 2008).

#### **4.6. Common Amodal Perceptual Representations for Thought, Action, and Learning**

What is the format of perceptual representations? A holdover from traditional theories is that information that comes in through sight is encoded in a visual representation, information gotten through hearing is encoded in an auditory representation, and so on. Products of different senses, if stored in separate encodings, would have to be subject to endless associations and calibrations to achieve even the simplest results in representing the world. When you perceive a bird that both squawks and flaps its wings, your brain would require complicated transactions to relate the squawking in the auditory world to the flapping in the visual world.

The idea that perceptual information must be saddled with the fragmentation of a separate visual world, an auditory world, a tactile world, etc. came originally from the obvious fact that we use different sense organs to pick up information, the unique sensations that characterize each sense,



and from the assumption that the contents of perception were aggregates of these sensations. Such an account leads irrevocably to the idea of separate representations in the separate senses, along with the need for associative or inference processes that have to be used to connect them.

If, as is now recognized, separate sensory input channels furnish more abstract information about structure in the world, it would make sense that, at least to some degree, these outputs converge into a common representation. Unlike the ideas that perception is amodal and information based, this idea is not yet a consensus view; it is still common for researchers to discuss multisensory integration or amodal representations in distinct senses (Nanay, 2010; Pouget, Deneve, & Duhamel, 2002). Yet, along with the obvious functional utility of having perceptual descriptions in a common amodal representation, there is now considerable evidence for early and intrinsic connections across the senses (Falchier, Clavagnier, Barone, & Kennedy, 2002; Knudsen & Knudsen, 1985; Meltzoff & Moore, 1989; Spelke, 1987; Stein & Meredith, 1993; Wertheimer, 1961).

Some neurophysiological evidence directly implicates encoding of amodal properties, such as location, apart from particular sensory channels. Knudsen (1982) discovered cells in the optic tectum of barn owls that respond to locations in space, whether specified auditorily or visually. This is direct evidence for a system encoding information about space and time, into which sensory channels feed, rather than a set of separate sensory representations. Much recent work in a variety of mammalian species also suggests that the brain is wired to connect the sensory input channels much earlier than was previously understood. Even early cortical areas, such as V1 and A1, that have been considered exclusively involved with one sense, have been shown to have multisensory influences (Falchier et al., 2002; Ghazanfar & Schroeder, 2006; Stein & Meredith, 1993). Stein and Stanford (2008, p. 263), in reviewing an extensive neurophysiological literature, conclude that "...evidence of early multisensory convergence raises fundamental questions about the sensory-specific organization of the cortex" and "These observations question whether there are any exclusive, modality-specific cortical regions and, thus, whether it is worth retaining designations that imply such exclusivity."

A wide variety of evidence and argument supports the idea that to support learning, thought and action, perceptual descriptions must involve a common, amodal representation, rather than merely modality-specific records (Ernst & Banks, 2002; Klatzky, Wu, & Stetten, 2008; Lehar, 1999; Stoffregen & Bardy, 2001).

#### 4.6.1. Embodied Cognition

Another important question is whether these representations must always be tied to *actions*. The idea of embodied cognition is a relatively recent set of ideas that suggests a close relationship between perception, cognition, and action. Like Barsalou's PSS approach, embodied cognition views tend to deny the idea of abstract cognitive representations separate from episodes of perceiving and acting. Thelen (2000) expresses the idea this way:

*To say that cognition is embodied means that it arises from bodily interactions with the world and is continually meshed with them. From this point of view, therefore, cognition depends on the kinds of experiences that come from having a body with particular perceptual and motor capabilities ... (p. 5)*

One issue in evaluating embodied cognition views is that there are a variety of them. Wilson (2002) has identified at least six possible basic claims of embodied cognition. They are:

1. Cognition is situated. Cognitive activity takes place in the context of a real-world environment, and it inherently involves perception and action.
2. Cognition is time pressured. We are "mind on the hoof" (Clark, 1997), and cognition must be understood in terms of how it functions under the pressures of real-time interaction with the environment.
3. We off-load cognitive work onto the environment. Because of limits on our information-processing abilities (e.g. limits on attention and working memory), we exploit the environment to reduce the cognitive workload. We make the environment hold or even manipulate information for us, and we harvest that information only on a need-to-know basis.
4. The environment is part of the cognitive system. The information flow between mind and world is so dense and continuous that, for scientists studying the nature of cognitive activity, the mind alone is not a meaningful unit of analysis.
5. Cognition is for action. The function of the mind is to guide action, and cognitive mechanisms such as perception and memory must be understood in terms of their ultimate contribution to situation-appropriate behavior.
6. Off-line cognition is body based. Even when decoupled from the environment, the activity of the mind is grounded in mechanisms that evolved for interaction with the environment—that is, mechanisms of sensory processing and motor control.

Claims 1 and 6 are perhaps the most significant for understanding representations, as well as the relation of PL to high-level cognitive tasks. We do not attempt any comprehensive analysis here, but limit ourselves to extending the important points derived earlier for understanding PL and cognition.

If the idea of embodied cognition is taken to mean that we do not have any abstract representations, able to be processed separately from the execution of actions, it is probably incorrect, and it would fail to allow a reasonable account of PL effects in high-level domains, for much the same reasons that plague the PSS and classical accounts. Specifically, the selective, abstract, and amodal properties of perceptual representations—the same ones that make the products of perception and PL most useful for complex cognitive tasks—preclude too close a coupling of PL with specific actions. As we will see below, evidence from PL interventions in high-level cognitive domains suggests that when learners come to apprehend important structures, this learning may improve their performance on a variety of tasks, including remote transfer tasks. Structure may be learned and used apart from specific actions. PL phenomena of this sort remind us of the classic work in animal learning indicating that stored representations obtained from perception can be used flexibly for different actions (Tolman, 1948), and, if we can add an update, for thinking. Binding perceptual representations too closely to specific actions would be problematic for reasons analogous to PSS ideas, where rerunning segments of prior perceiving episodes, complete with sensory activations, would seem to impede the extraction of abstract invariance detectable in new contexts. Just as Tolman argued for representations that could not be explained as stimulus–response pairings, embodiment consisting of a necessary connection between perceptual representations and specific actions would fail to provide a reasonable account of perception or PL. That said, many versions of embodied cognition, including most of the claims above, do not mandate such an extreme connection. Indeed, the general idea that advances in understanding may emerge from considering connections among perception, action, and thought is an idea with which we sympathize. For example, our argument regarding the use of spatial representations in symbolic domains such as mathematics might be considered to be related to several of the six claims considered by Wilson (2002).

#### **4.7. Implications for Perceptual Learning**

The classical view came with its own view of PL, because all of perception, as opposed to registration of raw sensations, was, in fact, associative learning. This view has been superseded by a generation of direct evidence about

perceptual development, indicating that perceptual systems deliver ecologically meaningful descriptions, even from birth (Bushnell, Sai, & Mullin, 1989; Held, 1985; Kellman & Spelke, 1983; Meltzoff & Moore, 1977; Slater, Mattock, & Brown, 1990; Walk & Gibson, 1961; for a review, see Kellman & Arterberry, 1998). The classic perceptual learning burden of constructing meaningful reality from associating sensations is obviated by an improved picture of early perception.

The revised view of perception as sensitive to information about important environmental properties comes with its own mandate for PL, however. An observer at any time is surrounded by a wealth of meaningful information about objects, surfaces, and events. There are an unlimited number of environmental features and relations that could be important for different tasks. Processes of learning serve to optimize performance of particular tasks by discovering which information is relevant to them, refining and attuning perceptual mechanisms to selectively extract this information, and automating that extraction (E. Gibson, 1969; Kellman & Garrigan, 2009). This kind of PL—that makes perceivers better at discovering and extracting currently available information—is the prevailing notion of PL in contemporary research.

Taken together, contemporary views of perception and PL provide clear foundations for beginning to understand and explore the role of PL in high-level cognitive tasks. The properties of perception that figure prominently are these: Perceptual representations are amodal, abstract, and selective. These are the properties that allow them to be functionally useful in thought and action. Extraction of complex relations connects directly to high-level thinking and underwrites action. If perceptual representations were not in a form that connects to capacities to reason, imagine, and plan, it would be hard to see their point. The synergistic relationship between extraction of important structure and thinking propels learning and the development of expertise.



## 5. PERCEPTUAL LEARNING AND INSTRUCTION

Mere mention of the word “instruction” evokes an image of teacher speaking to a class. Our ordinary intuitions about teaching and learning in formal settings, as well as most learning research, appear to be colored by a stereotype about what learning is and how it works. Bereiter and Scardamalia (1998) described this stereotype as a “folk psychology” view of learning, specifically, what they termed the “container metaphor”:

*Knowledge is most readily conceived of as specifiable objects in the mind, such as discrete facts, beliefs, ideas... (Learning) ... involves retaining and retrieving such objects. (Bereiter & Scardamalia, 1998, p 487).*

As we have seen, PL encompasses much that falls outside of this view of learning. Bereiter and Scardamalia contrasted with the conventional “mind as container” view a different idea: “mind as pattern recognizer.” PL is the type of learning that leads to mind as pattern recognizer.

That changes in the way information is extracted are important to expertise has been frequently documented. De Groot (1965), himself a chess master, studied chess players, with the expectation that master level players considered more possible moves and countermoves or in some sense thought more deeply about strategy. Instead, he found that their superiority was shown primarily on the perceptual side. Masters had become able to extract meaningful patterns in larger chunks, with greater speed and less effort than less skilled players. De Groot (1965) suggested that this profile is a hallmark of human expertise in many domains:

*We know that increasing experience and knowledge in a specific field (chess, for instance) has the effect that things (properties, etc.) which, at earlier stages, had to be abstracted, or even inferred are apt to be immediately perceived at later stages. To a rather large extent, abstraction is replaced by perception, but we do not know much about how this works, nor where the borderline lies. (pp. 33–34)*

Similar differences between experts and novices have since been found in research on expertise in a variety of domains, such as science problem solving (Chi, Feltovich, & Glaser, 1981; Simon, 2001), radiology (Kundel & Nodine, 1975; Lesgold, Rubinson, Feltovich, Glaser, & Klopfer, 1988), electronics (Egan & Schwartz, 1979), and mathematics (Robinson & Hayes, 1978). An influential review of learning and its relation to education (Bransford, Brown, & Cocking, 1999) summed it up this way:

*Experts are not simply “general problem solvers” who have learned a set of strategies that operate across all domains. The fact that experts are more likely than novices to recognize meaningful patterns of information applies in all domains, whether chess, electronics, mathematics, or classroom teaching. In De Groot’s (1965) words, a “given” problem situation is not really a given. Because of their ability to see patterns of meaningful information, experts begin problem solving at “a higher place” (DeGroot, 1965). (p. 48)*

## 5.1. Natural Kind Learning

It is interesting that school learning centers so heavily on explicit verbal instruction about facts and procedures, given that more implicit PL

processes appear to dominate the prodigious learning accomplishments of children in the years before they reach school age. Much of early learning may be characterized as discovery processes in PL, and these include apprehension of highly abstract relations, even very early on (Marcus, Vijayan, Bandi Rao & Vishton, 1999). Natural kind learning exemplifies some of the most interesting and powerful characteristics of this kind of learning and transfer. Imagine a young child going for a walk with her father. Upon seeing a dog, the child points, and her father says "That's a dog." Suppose this particular dog is a small white poodle. On some other day, the child sees another dog—this one a large black Labrador retriever. Again, someone says "dog." And so on. With each instance, something about a particular dog (along with the label "dog") is encoded. As the process continues, and a number of instances (probably not a particularly large number) have been encountered, the child becomes able to look at a new, never before seen dog and say "dog." This is the magical part, as each new dog will differ in various ways from *any* of the examples previously encountered. Moreover, the child is concurrently coming to distinguish correctly novel instances of dog, cat, squirrel, etc., from each other. A particular cat or squirrel may have properties that resemble some known dog; a small black dog and a large black cat are more similar in color and size than are a large black and small white dog. Despite similarities of instances across different classes and differences among instances within classes, the learner comes to extract properties sufficient to classify novel instances accurately. Much of the relevant PL would seem to require discovery of abstract relations, as simple features, such as color, are seldom the crucial determinants. Shape variables are often important, such as the differing jaw or body structures of dogs and cats. Shape variables are highly relational and abstract, rather than tied to particular colors, sizes, and contexts, which is what allows those who have undergone this kind of learning to effortlessly recognize a glass tabletop ornament as a dog versus a cat.

The properties underlying a classification can be complex and implicit. If a child, or even an adult, is asked to state a set of rules that would allow a novice to distinguish dogs, cats, and wolves, they cannot ordinarily do so. Even the hypothesis about jaw and body structure of dogs and cats, mentioned in the previous paragraph, is a conjecture the authors have generated from poring over examples. For adults, even learning researchers, knowing cat versus dog when one sees them is easy, but furnishing an account in declarative knowledge or a diagnostic procedure is hard, and it is not a typical accompaniment of the ability to recognize.

Nor do the toddler's striking feats of natural learning occur from being given lectures on the distinguishing features of dogs or cats. Rather, structure is extracted from encountering instances and receiving category feedback. Such PL processes are crucial not only for developing understanding of the objects and events in the world; they also play a pivotal role in language acquisition, at multiple levels. Concepts like noun, verb, adverb, and preposition are taxing enough when taught explicitly in middle school. How is it that these abstract classes are extracted and used in language acquisition, allowing grammatical structures to be processed (e.g. Hirsh-Pasek & Golinkoff, 1997) and facilitating the learning of new words? At a different level, learning may be involved in the ability of the young language learner to detect invariance in the structure of speech signals across different speakers. Evidence suggests that the PL processes needed for these achievements, including clear cases of abstract PL, are present relatively early in infancy (Gomez & Gerken, 1999; Marcus et al., 1999; Saffran, Loman, & Robertson, 2000).

## 5.2. Relations among Types of Learning: Toward a "Fundamental Theorem of Learning"

When a child starts school or other formal learning, the focus of most instructional efforts, as it has been in most research on instruction, is on declarative and procedural activities. This emphasis can be seen, in part, as fitting with important patterns that scientists have discovered regarding human cognitive development. Before a certain age, the introduction of formal concepts and reasoning is likely to be pointless (NRC, 2001; Piaget, 1954).

Conversely, PL is among types of learning that seem to operate from the beginning of life, and it plays an important role in natural kind learning, language acquisition, and transactions with many kinds of objects and events. When a child has reached school age, it might be assumed that with those foundations already in place, "higher" cognitive activities—encompassing explicit facts, concepts, procedures, and thinking—take center stage.

We believe it would be a misunderstanding, however, to believe that when more explicit aspects of learning are introduced, the PL components of learning fade into the background. Although we do not attribute this view explicitly to anyone, it may be natural to assume that by school age, perceptual transactions with the environment have been largely mastered or that they operate in a relatively steady-state fashion. A related point may

be made about the content of thought and learning. Theories of cognitive development have tended to be saturated with classical views of perception (Kellman & Arterberry, 1998); thus, in Piaget's views, and subsequent related views, the early role of the senses is in associative "sensorimotor" transactions. Conceptual inputs in various domains must operate to make perceptual data useful for abstract knowledge (e.g. Leslie, 1995; Mandler, 1988, 1992; Piaget, 1952, 1954). Research that has produced a radically different understanding of the perceptual starting points of development changes this picture and has profound implications for cognitive development, which have been discussed elsewhere (e.g. Jones & Smith, 1993; Kellman & Arterberry, 1998). In the present context, the crucial consequences of the contemporary understanding of perception as delivering abstract structural knowledge are that 1) the perceptual part of learning remains important in most or all learning domains, and 2) the products of perception are not static or previously mastered, but are dynamically changing as an important part of learning in any domain.

Perhaps most interesting and important, the changes in perceptual pickup and the use of declarative and procedural knowledge and reasoning should not be considered unrelated aspects of task performance. There is a crucial, interactive relationship between these, one that parallels the close coupling of perception and action (J. Gibson, 1966, 1979). Although it has seldom been emphasized in learning research, PL processes—that attune the encoding, classification, discrimination, or recognition of incoming information—bear a pervasive relationship to the better-known declarative and procedural aspects of learning. Only half jokingly, we call this the "Fundamental Theorem of Learning." It states that

*All effective use of declarative and procedural learning presupposes pattern recognition.*

Suppose a learner in some domain has acquired a vast array of facts, concepts, and procedures. How are these deployed? How do they lead to effective problem solving in new situations as they arise? Randomly producing facts and procedures is at best inefficient and at worst pathological. Obviously, facts and procedures must be used selectively and appropriately. Accomplishing appropriate selection depends on accurate classification of problems or situations. When one is confronted with a new problem or situation, which facts apply? Which procedures are relevant? Fundamentally, these are questions of encoding and classifying the input; they require recognizing, amidst irrelevant detail, the structural patterns that matter. They are pattern recognition problems.



Becoming able to see what matters in a given situation has long been regarded as the essence of meaningful learning and creative problem solving (c.f. [Duncker, 1945](#); [Wertheimer, 1959](#)). What has often been missing from the discussions of the role of seeing in problem solving is the learning process that allows the learners to become able to recognize, in new situations, the meaningful structures that matter and to distinguish the relevant from the irrelevant. This is the role of PL, and our statement of this “fundamental theorem” is simply a reminder that even in high-level learning tasks and domains, processes that advance encoding, discrimination, classification, and structure recognition allow facts, concepts and procedures to be used effectively.

### 5.3. Perceptual Learning Technology

Modeling PL is a complicated and unfinished effort ([Ahissar & Hochstein, 2004](#); [Fahle & Poggio, 2002](#); [Kellman & Garrigan, 2009](#); [Petrov et al., 2005](#)). This is especially true for perceptual classifications that are based on abstract relations ([Kellman, Burke, & Hummel, 1999](#); for discussion, see [Kellman & Garrigan, 2009](#)). There are currently relatively few models that even purport to discover abstract relationships that govern a classification, even in restricted domains. Improving our understanding of such abilities will be valuable for many scientific and technological reasons. For example, in computer vision and artificial intelligence, we still lack systems that can learn to recognize cats in ordinary scenes, much less learn to classify a glass table ornament as a cat, and we are far away from being able to extract even more abstract regularities, such as when a tone of voice conveys sarcasm.

Fortunately, the task of understanding the conditions under which PL occurs and the variables that affect it is a much more tractable one than developing models of high-level PL. Understanding the principles of PL is an active area of research (e.g. [Ahissar & Hochstein, 2004](#); [Mettler & Kellman, 2010](#); [Seitz & Watanabe, 2005](#); [Zhang et al., 2010](#)).

Some efforts have focused on complex, real-world tasks, attempting to systematically address PL and accelerate the growth of perceptual expertise in instructional settings. These efforts have already produced remarkably successful outcomes in a variety of learning domains.

[Kellman and Kaiser \(1994\)](#) developed PLMs to address difficult problems in aviation training. In a Visual Navigation PLM, pilots learned navigational skills by mapping, on short, speeded trials, videotaped segments of out-of-the-cockpit views onto locations shown on standard visual navigation (Visual Flight Rules sectional) charts. Remarkable improvements in

accuracy and speed occurred in less than an hour of training, even among experienced aviators. In an Instrument Flight PLM, the focus was on flight instrument interpretation. On short speeded trials, pilots classified aircraft attitude (e.g. climbing, turning) from an array of primary flight displays used by pilots to fly in instrument conditions. They found that under an hour of training allowed novices to process configurations more quickly than and just as accurately as civil aviators who had on average 1000 h of flight time (but who had not used the PLM). When experienced pilots used the PLM, they also showed substantial gains, paring 60% off the time needed to interpret instrument configurations.

PL interventions to address speech and language difficulties have been shown to produce benefits (Merzenich et al., 1996; Tallal, Merzenich, Miller, & Jenkins, 1998). For example, Tallal et al. showed that auditory discrimination training in language learning using specially enhanced and extended speech signals improved both auditory discrimination performance and speech comprehension in language-impaired children.

Applications in medical and surgical training illustrate the value of PL in addressing dimensions of learning not encompassed by ordinary instruction. Guerlain et al. (2004) applied PLM concepts to address issues of anatomic recognition in laparoscopic procedures. They found that a computer-based PLM approach patterned after the work of Kellman and Kaiser (1994) produced better performance than traditional approaches. The training group presented with variation in instances selected to encourage learning of underlying invariance later showed improvement on perceptual and procedural measures, whereas a control group who saw similar displays but without the structured PLM did not. Their data implicated PL as the source of the improvement, as neither group advanced on strategic or declarative knowledge tests.

More recently, Krasne et al. (submitted) developed and tested two computer-based perceptual/adaptive learning modules (PALMs) in the preclerkship curriculum for all first- and second-year medical students at the UCLA School of Medicine. One module focused on pathologic processes in skin histology images (*Histopathology PALM*) and the other for identifying skin-lesion morphologies (*Dermatology PALM*). The goal was to assess students' ability to develop pattern recognition and discrimination skills leading to accuracy and fluency in diagnosing new instances of disease-related patterns. These were short learning interventions, with objective learning criteria typically achieved in 15–35 min. Results indicated strong learning gains in accurately classifying previously unseen cases, elevating students' performance

in both the first and second years of medical school well beyond the levels attained from conventional instruction alone. There were strong gains in both accuracy and fluency; besides becoming more accurate, learners averaged a 53% reduction in classification time across both years and PALMs. Effect sizes averaged in the 1.0–1.5 range for both accuracy and fluency. These results with brief interventions suggest that PL interventions impact aspects of learning that are not well addressed by conventional instruction. They also suggest remarkable promise for the use of PL to improve learning in a variety of medical and other domains.

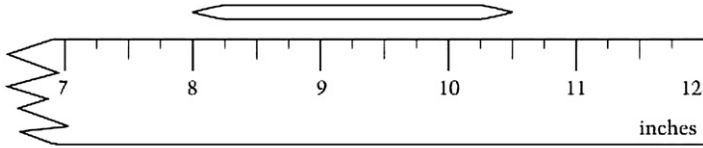
Over the past decade, we have undertaken large-scale, systematic efforts to study and apply PL technology in mathematics and science learning (Kellman et al., 2009; Massey et al., 2011; Silva & Kellman, 1999; Wise et al., 2000). Although these subjects involve a variety of cognitive processes, they rely substantially on pattern recognition and fluent processing of structure, as well as mapping across transformations (e.g. in algebra) and across multiple representations (e.g. graphs and equations). Few instructional activities directly address these aspects of learning, and a variety of indicators suggest that they may be disproportionately responsible for students' difficulties in learning (Kellman et al., 2009). Findings consistently indicate that even short PLM interventions can accelerate fluent use of structure in contexts such as the mapping between graphs and equations (Kellman et al., 2008; Silva & Kellman, 1999), apprehending molecular structure in chemistry (Russell & Kellman, 1998; Wise et al., 2000), processing algebraic transformations, and understanding fractions and proportional reasoning (Kellman et al., 2008, 2009; Massey et al., 2011). Earlier, we presented the example of an *Algebraic Transformations PLM*. To convey the scope and approach of PLMs in mathematics learning, we describe here one other PLM in detail and summarize some others. These examples will help to illustrate both the learning effects from PLMs as well as their distinctive features as learning interventions.

An illuminating example is a PLM that we developed to help elementary students master linear measurement with rulers of varying scales. When one considers a standard ruler, it is a rather remarkable device that organizes a numerical symbol system in a spatial layout—essentially the positive side of a rational number line on a strip of wood or plastic. The continuous extent is evenly partitioned into units and marked by numbered hash marks, with hash marks of several sizes arranged to indicate different scales layered on the same ruler (e.g. half inches, quarter inches, eighth inches). Once one has acquired expertise with this instrument, it is a simple matter to “just

see” the structure. The inches or quarter inches or centimeters or meters are readily perceived as objects that can be manipulated and enumerated in various ways to measure linear extents.

As countless teachers can testify, however, acquiring such insight is not a simple or reliable achievement for many elementary or even middle school students, despite conscientious instruction. An indication of the learning difficulty comes from results on the National Assessment of Education Progress (<http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics>), on a released item known as the broken ruler problem. A version of the problem is illustrated in Figure 4.3. A toothpick is pictured above a standard 12-inch ruler that has been broken so that the left-hand edge starts at 7 inches. The toothpick is positioned so that it starts at 8 and ends at  $10\frac{1}{2}$ , and students are asked to enter the length of the ruler. Alarming, only 20% of 4th graders and 58% of 8th graders give the correct answer. Of particular interest are the two most common incorrect answers:  $10\frac{1}{2}$  and  $3\frac{1}{2}$ . Students who give the former answer are most likely following a poorly understood, inflexible procedure that involves reading the rightmost endpoint as the length—simply ignoring that the ruler is broken. Students who say that the toothpick is  $3\frac{1}{2}$  inches long are probably relying on a counting routine and counting the hash marks starting with the leftmost hash mark as “1.” (It is a common classroom observation that students are extremely puzzled as to why the left-most edge of a ruler is 0 rather than 1, and why they are told to line things up starting at 0. After all, when counting discrete items, one always starts with one, not zero.)

Both of these incorrect answers indicate that the students are not perceiving units on the ruler that have extent. From conventional instruction, they have picked up some aspects of measurement facts and procedures, but the mapping of what they have learned onto structure in the problem is faulty. They do not recognize that an inch (or centimeter, etc.) on a ruler is the extent between the hash marks that demarcate the unit, not just the point where the numbered hash mark is located. The beauty of the broken ruler problem is that it reveals this; students succeed with much higher accuracy if they are given an ordinary ruler problem, in which the zero point lines up with the left edge of the toothpick. A related and persistent difficulty is that students struggle to map fractions to rulers. Difficulties with fraction notation aside, if a student does not see an extended unit to begin with, he or she will have difficulty identifying the subpartitions of units that map to fractional quantities.



What is the length of the toothpick in the figure above?

**Figure 4.3** The “broken ruler” problem. Released measurement item from the 2003 National Assessment for Educational Progress (US Department of Education, IES, National Center for Education Statistics, <http://nces.ed.gov/nationsreportcard/itmrlsx>).

To address this problem of seeing the relevant structure, we developed learning software that presents students with many short, interactive, animated learning trials in which students interact with the key structures and relationships underlying linear measurement. A typical trial presents the student with a graphic display showing a ball on top of a ruler and billiard cue poised to strike it. The student is given either a starting point and an ending point and asked to say the distance traveled, or they are given a starting point and a traveling distance and are asked to say what the endpoint will be. The learning items in the database vary with the types of values involved, whether the rulers are fully versus partially labeled, and whether they are partitioned in the most economical way to solve the problem or are overpartitioned (e.g. a ruler marked in units of sixteenths for a problem involving eighths). Movement on the ruler can be either rightward or leftward. The quantities involved vary from single digits into the hundreds and included both fractions and integers. Learners receive immediate animated feedback on each trial and are also given periodic feedback on their progress through the module.

Instead of emphasizing verbal explanations or procedural calculations, this *Linear Measurement PLM* concentrates the students’ attention and effort on learning to pick up relevant structures and relationships. The items in the learning set are designed so that each student sees many varied examples; these are conditions in which PL processes come to discover and fluently extract important structures in different contexts. In this way, PLMs accelerate students’ expertise until they are able to “just see” what is important and relevant in each problem.

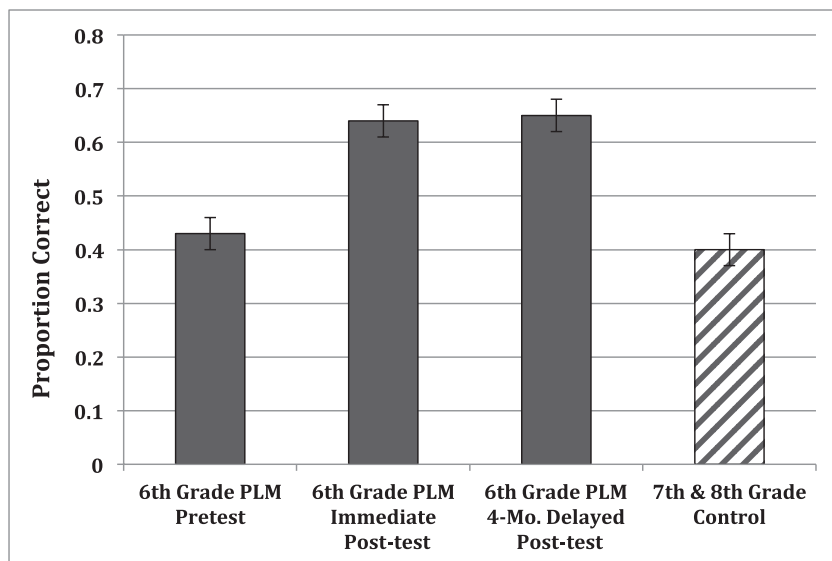
In a formal study, 63 sixth-grade students in a low performing urban middle school completed a pretest and used the *Linear Measurement PLM*,

then took an immediate post-test as well as a delayed post-test a full 4 months later. The 6th graders were compared with a group of forty-nine 7th and 8th graders in the same school who took the assessment without using the module. The assessment, which included many transfer items that did not resemble the learning trials, tested children's ability to use a partitioned number line to express the length of a line segment in generic units; to use both conventional and broken rulers to measure lengths in inches and centimeters; to use conventional and broken rulers to construct extents with varying lengths; to solve addition and subtraction problems with fractions; and to solve open-ended word problems involving linear measurements. Both the 6th grade intervention students at pretest and the older control students scored <50% on the assessment. After completing the module, the 6th graders' scores improved dramatically (Figure 4.4), with effect sizes (Cohen's  $d$ ) comparing pretest scores versus post-test scores and intervention versus control groups ranging from 0.86 to 1.06 (Kellman, Massey & Son, 2009; Massey, Kellman, Roth, & Burke, 2011). The studies also demonstrated remarkable durability of learning: Scores on delayed post-tests conducted 4 months later, with no intervening study activities, indicated that the learning gains for the intervention groups were fully maintained.

Other PLM interventions in mathematics learning have produced comparable results. In the area of fractions and measurement, PLMs focusing on partitioning and iterating units and mapping equivalent quantities across different units not only produced effect sizes in the range of 1.0 to 2.8 but led to remote transfer of learning to multiplying and dividing fractions and mixed numbers (tasks that were not part of the PLM). Moreover, as in the case of the *Linear Measurement PLM* described above, the learning gains showed no decrements in delayed post-tests administered 4–5 months later. Both the remote transfer and durability of the learning highlight important characteristics of PL: Becoming able to see relevant structure in a domain allows that structure to be used in varied tasks and comprises an enduring kind of learning.

#### 5.4. Elements of Perceptual Learning in Instruction

These examples of PLMs in real-world learning contexts illustrate some of the conditions that produce PL effects and some of the characteristics of learning attainments from these interventions. More generally, what are the elements of PL interventions?



**Figure 4.4 Results from a Study of the Linear Measurement PLM.** Pretest, immediate post-test, and delayed post-test scores are shown for the 6th grade intervention group compared to 7th and 8th grade students in the same school. The delayed post-test was administered after a delay of 4 months, with no intervening study activities. Error bars indicate  $\pm 1$  standard error of the mean (Adapted from Kellman, Massey & Son, *TopICS in Cognitive Science*, 2009; Cognitive Science Society, Inc., p. 16).

Kellman et al. (2009) argued that at least three general properties are crucial.<sup>3</sup> The most basic requirement is that PL tasks focus on the extraction of structure. PLMs involve encoding, discrimination, comparison, and/or classification. A contrast in mathematics learning is that PL interventions need not involve computation of numerical answers. In PL tasks, the learner engages in practice with displays or representations in which success depends on the learner coming to attend to, discriminate, classify, or map structure. Utilizing structure is of course involved in other types of instruction, but

<sup>3</sup>This section focuses on characteristics that define PL interventions as a distinctive type of learning activity. Many more specific features of PLMs, not discussed in detail here, serve to optimize learning in this general format and to configure them for particular learning challenges and goals. These include issues of sequencing, feedback, variation of positive and negative instances of categories, mixing of learning tasks, integration of PL activities with conventional instruction, and so on. A number of features of PL technology and related adaptive learning technology are covered by US patent #7052277 and patents pending, assigned to Insight Learning Technology, Inc. For information, please contact the authors or [Info@insightlearningtech.com](mailto:Info@insightlearningtech.com).

a PL task focuses on commonalities and variations in structure as its primary learning content. A second characteristic is that PLMs tend to involve numerous short classification trials with varied instances. The learner makes classifications on these trials and (in most cases) receives feedback. Systematic variation across learning instances is crucial, because in most real-world tasks, PL involves the discovery of invariance amidst variation (Gibson, 1969). Discovery processes require sufficient variation for relevant properties to be disentangled from irrelevant ones. This aspect of PL interventions is most powerful in producing transfer of learning to new situations that involve common or related structures. Emphasis on the discovery of invariance amidst robust variation is crucial in realistic learning tasks, but it differs from most contemporary laboratory studies of PL, which typically target simple sensory discriminations and involve large numbers of trials with a small set of fixed stimuli (e.g. Fahle & Poggio, 2002; for a discussion, see Garrigan & Kellman, 2008). Finally, PL interventions tend to have minimal emphasis on explicit instruction. The learning comes from transactions with the input, not verbal exchanges. The primary task in a PL intervention does not involve verbal or written explanations of facts, concepts, or procedures. This is a major difference from conventional instruction, which is dominated by explicit description (which also addresses important aspects of learning). PL interventions may incorporate explicit introductions or brief discussions, but these do not comprise the central learning tasks nor are they capable of producing the results obtained with PLMs.

Another important question is, “How do we know that PL effects are occurring from a learning intervention?” In complex tasks and realistic learning settings, we have less control over materials and activities than in most laboratory situations. Moreover, we would expect, as we have argued in this chapter, that PL works synergistically with other processes of learning and thinking in domains such as mathematics. Given these background conditions, it is unlikely that any intervention in a complex-learning domain targets PL uniquely, and it is difficult to claim that any learning gains are solely the result of PL. Likewise, although the issue has not received much attention, it would be hard to claim that effects produced by other instructional interventions do not involve a PL component. PLMs attempt to condense or accelerate PL, but PL no doubt goes on less systematically in other learning situations.

Synergies aside, there appear to be some characteristic signatures of PL effects. Kellman et al. (2009) suggested four of these, summarized in Table 4.2.: 1. *Generativity in structure use*. PLMs in complex learning



domains are designed to improve pickup and processing of structural invariants across variable contexts. A hallmark of successful PL is the evidence of accurate and/or fluent classification of novel cases. Moreover, PLMs often facilitate remote transfer to different-looking problem types that involve the same underlying structure. Such transfer is a notorious problem in settings using conventional declarative instructional approaches. Evidence of accurate and fluent classification of novel instances, and transfer to contexts involving different procedural requirements but common structures, provides evidence of PL. 2. *Fluency effects.* PL effects typically include improved fluency of information extraction (indicated in measures of speed, parallel processing, or reduced effort or cognitive load). Acquisition data within PLMs suggest that fluency in information extraction increases gradually across interactive trials. Gradual improvement is not unique to PL but does contrast with some effects of declarative instruction, in which a learner may either know or not know a certain concept. A particularly clear case of PL effects on fluency can be made when relevant declarative knowledge is already present prior to an intervention, and a PL intervention produces improved fluency, as in the *Algebraic Transformations PLM* described

**Table 4.2** Some Possible Signature Effects of Perceptual Learning Interventions. The effects shown are common outcomes of PL interventions that tend to distinguish them from outcomes of instruction focused on declarative or procedural knowledge (see text).

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#### **Generativity in use of structure**

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- Accurate and/or fluent processing of novel cases
  - Improvement on unpracticed tasks that involve learned structures
- 

#### **Improvements in fluency**

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- Faster processing
  - Greater parallel processing
  - Reduced cognitive load or effort
- 

#### **Implicit pattern recognition versus explicit knowledge**

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- Improved performance without new explicit declarative or procedural knowledge
- 

#### **Durability of learning**

---

- Improved information extraction and structural intuition that persist over long delays and are highly resistant to forgetting
-

earlier. 3. *Implicit pattern recognition versus explicit knowledge.* Although PL may provide important scaffolding for explicit, verbalizable knowledge, PL itself need not involve changes in explicit knowledge. PL changes the way a learner views a problem or representation, and these changes need not be accompanied by new explicit facts, concepts, or procedures. Transfer tests routinely indicate this dissociation from PL interventions (Guerlain et al., 2004; Kellman et al., 2009). 4. *Delayed testing effects.* Common wisdom has it that one never forgets how to ride a bicycle. If true, riding a bicycle, a task that clearly involves considerable PL, differs from most declarative and procedural learning. It is not by accident that math teachers spend the first month of a new school year reviewing content from the prior year. Facts and procedures are subject to substantial forgetting over a period as long as a summer vacation from school. Although more research is needed, there are indications that the improved facility in picking up patterns and structure from PL, like riding a bicycle, may be less subject to decay over time. In the measurement and fraction PLMs described above, we have consistently observed no decrements in learning gains when students were tested after 4- to 5-month delays (Kellman et al., 2009; Massey et al., 2011).

It is also possible to test directly for PL effects. In domains where the central task is clearly focused on classification, such as the *Dermatology and Histopathology PALMs* described above, rapid and accurate classification of new instances illustrates straightforwardly that learners have improved in the pickup of information. For PL interventions in cognitive domains that also involve symbolic material and substantial reasoning components, the situation is more complicated in attributing learning gains to specifically PL effects. In applying PL technology to such domains, investigators have usually had as a first priority showing that PL leads to meaningful learning gains, beyond those of conventional instruction. Thus, tests of learning and transfer have typically assessed learning on important domain-relevant tasks; in mathematics PLMs, these have involved tasks such as solving algebra problems, performing operations with fractions or measurement, or generating a correct graph from an equation or an equation from a word problem (Kellman et al., 2008). However, more basic psychophysical tests in complex PL domains are also possible. Thai, Mettler, and Kellman (2011) showed that PL interventions, like those we have used in complex, symbolic domains, produce basic changes in information extraction. Participants were trained to classify Chinese characters, based on either overall configurations (structures), featural relations (components),

or nonrelational information (stroke count), used as a control. PLM participants showed strong domain-relevant learning gains in discriminating and classifying Chinese characters. Before and after training, however, they were also tested for basic changes in information extraction using a visual search task, which had not been part of training. Search displays contained all novel exemplars, involved manipulations of target-distractor similarity using structures and components, and included heterogeneous and homogeneous distractors. Robust improvements in visual search for structure and component PL training were found relative to a control group that did not undergo PLM training. These results provide direct evidence that high-level PL interventions improve learning by altering extraction of information, including changing perceptual sensitivity to important relational structures. This study is interesting in connecting a high-level cognitive task to changes in information pickup detectable by more basic psychophysical methods. Further research of this sort may prove useful, both in understanding the synergies of various cognitive abilities and in optimizing PL interventions.

Another significant issue for further research is how PL interventions might best be combined with other modes of instruction. Acquiring declarative and procedural knowledge, improving critical thinking, and other aspects of learning do not become less important because we are coming to understand that neglected PL components are crucial in many learning domains. In fact, it seems likely that instructional methods of all types will benefit from understanding more clearly these different components of learning and their interactions. A discussion of explicit concepts, or a proof, may be easier when the teacher knows that the student is correctly mapping the words onto problem structure, and a procedure may be better understood, better remembered, and certainly better applied, when the student can see where and why it applies.



## 6. CONCLUSION

Research in PL offers previously unsuspected synergies with high-level cognitive tasks and processes. Through an emerging technology of PL, it also promises remarkable potential to improve learning in almost any domain, including complex, symbolic ones. To understand and utilize these possibilities fully requires making clear basic connections between perception, cognition, and learning, especially the implications of contemporary views of perception as abstract, amodal, and selective. In this chapter, we

have tried to describe these connections in ways that allow us to integrate and illuminate recent research and applications of PL. We hope these efforts contribute to future progress in understanding cognition, perception, and learning.

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