Correspondence

Group Path Formation

Robert L. Goldstone, Andy Jones, and Michael E. Roberts

***Abstract*—When people make choices within a group, they are frequently influenced by the choices made by others. We have experimentally explored the general phenomenon of group behavior where an early action facilitates subsequent actions. Our concrete instantiation of this problem is group path formation where people travel between destinations with the travel cost for moving onto a location inversely related to the frequency with which others have visited the location. We compare the resulting paths to optimal solutions [minimal Steiner trees (MSTs)] and the "Active Walker" model of pedestrian motion from biophysics. There were systematic deviations from beeline pathways in the direction of MST. These deviations showed asymmetries (people took different paths from A to B than they did from B to A) and varied as a function of the topology of the destinations, the duration of travel, and the absolute scale of the world. The Active Walker model accounted for many of these results, in addition to correctly predicting the approximate spatial distribution of steps.**

***Index Terms*—Conformity, culture, group behavior, group choice, minimal Steiner trees (MSTs), path dependence, route formation, stigmergy, trail systems.**

I. Introduction

"Traveler, there is no path. Paths are made by walking."
Antonio Machado

Complex adaptive-system models are typically applied to natural phenomena, such as the pattern of stripes on zebras or seeds on a sunflower [2]. However, these modeling tools are also illuminating for understanding group behavior [3]. A complex-systems perspective can liberate us of our customary habit of focusing on individual behavior and instead encourage attention to emergent social organizations at a higher level than the individual [4], [5]. Social phenomena such as rumors, the emergence of a standard currency, transportation systems, the World Wide Web, resource harvesting, crowding, and scientific establishments arise because of individuals' beliefs and goals, but the eventual form that these phenomena take is rarely the goal of any individual.

The current research explores spatial group-choice behavior. When people make choices within a group, they are frequently influenced by the choices made by others ([6]; see [7] for a review). In particular, there is a striking similarity of choices and behaviors by people in a group, as is revealed by any casual observation of high schools, bars, or scientific meetings. One possible cause of this group-level similarity is that early choices by group members change the environment and, hence, the attractiveness of choices for subsequent group members. Many times, initial pioneers reduce the costs for followers who pursue similar paths. For example, recent cognitive neuroscience researchers can perform functional magnetic resonance imaging (fMRI) experiments rapidly and efficiently because of prior researchers' developments of brain imaging. Much, if not all, of what we know of as culture is built up in this fashion—by following and extending the innovations of predecessors [8].

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II. Determinants of Similarity Within a Group

There are several reasons why people in a group may behave similarly. People in a group may simply be similar to one another, which may be why they formed the group in the first place. Across many kinds of groups, including religious, special interest, occupational, and recreational communities, similar people tend to flock together [9], [10] and stay together once they have collected together [11].

A second reason for similarity within a group is that humans are uniquely adept at adopting one others' innovations. Cultural identity is largely due to the dissemination of concepts, beliefs, and artifacts across people. Our capacity for imitation has been termed "no-trial learning" by Bandura [12], who stressed that people perform behaviors that they would not have otherwise considered by imitating one another. Imitation is commonly thought to be the last resort for dull and dim-witted individuals. However, cases of true imitation are rare among nonhuman animals [13], requiring complex cognitive processes of perception, analogical reasoning, and action preparation. When combined with variation and adaptation, imitation is one of the most powerful methods for quick and effective learning. Learning theory has historically stressed individual learning in solitary conditions. However, there is a growing realization that learning and thinking, in general, are inherently social activities [14].

Social psychologists have conducted a considerable amount of research pursuing a third and related possibility: That people are motivated to conform to the group in order to curry the group's favor [15]. In social psychology, there has been a long and robust literature on conformity in groups [7], [16]. To some degree, conformity is found because people desire to obtain social approval from others [17]. For example, sometimes, when people give their answers privately, they are less likely to conform to the group's opinion than when responding publicly [15]. However, at other times, the conformity runs deeper than this, and people continue to conform to the group's opinion even privately [16]. In Ash's [18] classic experiments on conformity, subjects judged unambiguous stimuli after hearing other opinions offering incorrect estimates. Of his subjects, 69% conformed to the bogus majority. Milgram's [19] subsequent studies of conforming to authority dictates were directly inspired by Ash's studies.

There is considerable work on how individuals conform or use information provided by others. Field work also explores actual small groups of people engaged in cooperative problem solving [3]. However, there is less work with laboratory-controlled conditions that explores the dynamics of a group of participants solving problems as they exchange information. One related study is Latané and L'Herrou's [20] exploration of participants' sending e-mail messages to each other [6], as they tried to predict which of two options their group would select. Over the course of message exchanges, neighboring participants in the network tended to adopt similar choices, but there was also a continued diversity of choices across the entire network.

The adoption of others' ideas has been a major field of research not only in social psychology but also in economics, political science, and sociology. It is common in models of collective action to make an individual's decision to participate based upon their expectation of how many other people will participate [21]. A common outcome of a collective "I'll do it if you do it" mentality is for "tipping points" to arise in which adding a few more participants to an action leads to a positive feedback cycle in which still more participants sign on, leading to an exponential increase in participation for a time [22]. This behavior is a sensible policy, both because the likelihood of success of an innovation depends upon its public adoption rate [23] and because other people may have privileged information unavailable to the individual making a choice. The potential cost of this bandwagon behavior is wasted time, money, and effort in adopting new innovations [24], [25]. At the group level, another problem with bandwagons is that the full range of possible solutions is not well explored if the population is overly homogeneous.

III. Cultural and Technological Advances by Building Upon Pioneer Efforts

The work reviewed in the previous section indicates that people conform to other group members because perceived group pressure forces the individual to yield under the threat of rejection or the promise of reward ("normative conformity" [11]) or because they believe that others may know something that they do not ("informational conformity"). Although we certainly do not deny the strong role of conformity on people's behavior, we would like to suggest that, often times, the members of a community converge upon similar behaviors because of a beneficial and sensible propensity to take advantage of others' efforts. In particular, later choosers can take advantage of the efforts of earlier choosers. There are often intrinsic advantages to choosing options that are popular. When Vertical Helical Scan (VHS) became more popular than Beta as a format for video recording, then wise people chose VHS, because the popularity of VHS led to more movie titles being released on VHS. Similarly, Microsoft's popularity creates a positive feedback loop creating still further popularity. Exchanging files and obtaining software is relatively easy for Microsoft's users, ensuring its further popularity. A similar popularity advantage explains why the QWERTYUIOP keyboard continues to be chosen by most people despite its demonstrated inferiority to other keyboard arrangements.

In many business and economic models, the early entrant to a market has a large influence on setting the tone and agenda for the future of the market. Early cooperation or defection choices in social-dilemma games set the tone for subsequent play [26]. Early entrants to business sectors [27] and resource pools [28] shape the territory for future participants. Early attitudes and behaviors within a culture create norms that tend to be continued for subsequent generations, even after their extrinsic fitness value is no longer favorable [29]. Cultural norms can become dominant because members of the culture come to expect them. A very general phenomenon is for early choices to create precedents that are followed and extended by later choices.

Moore's law is a particularly striking case of this, where for the last 40 years, there has been a constant-rate exponential increase in the number of transistors per centimeter of integrated circuit, largely because technological advances pursue paths of previous innovations and extend them. Progress is achieved by exploiting and building upon previous discoveries. The specific concrete instantiation of this that we will be experimentally exploring is the evolution of spatial path systems.

IV. Engineering Considerations of Spontaneous Trail Formation

Our experimental instantiation of successors being influenced by predecessors' paths is the evolution of spatial path systems. Early trail blazers through a jungle use machetes to make slow progress in building paths, progress that is capitalized on and extended by later trekkers, who may then widen the trail, then later put stones down, then gravel, and then asphalt. Our specific experimental question is: What kind of trail systems do people spontaneously produce when they are motivated to take advantage of the trails left by others and, in the process of so doing, further reinforce these trails?

Finding principles that govern the spontaneous formation of people's paths is important for engineering systems for several reasons. First, an understanding of spontaneously emerging trails would allow the beneficial prediction of emerging path networks for specific applications. Architects, city planners, traffic engineers, and park designers are responsible for anticipating and guiding the movement of people in space. If a computational model can be developed that predicts eventual path systems, then parameters of the model can be manipulated to explore counterfactual "what if" scenarios and forecast the impact of physical interventions. Second, if the properties of spontaneously created path systems are understood, then it may be possible to employ them as alternatives to top–down architectural structures. The architectural community is generally reluctant to relinquish control of the design process for public spaces. However, if spontaneously created paths can be shown to have desirable properties, in terms of their resilience, flexibility, and efficiency, then some of this resistance could be overcome. Third, principles of geographic trail formation may be applicable to more abstract trails. Systems like the World Wide Web, peer-to-peer networks, and the Internet are not created from top–down control, but rather are self-organized structures that arise from local and regional needs to transmit information efficiently from starting to destination locations [30]. Principles gleaned from the concrete case of spatial trail formation can potentially be applied to the construction of less transparent and more abstract path systems.

A very general principle that can likely be transferred from concrete to abstract self-organized path formation is stigmergy. Stigmergy is a form of indirect communication between agents that is achieved by agents modifying their environment and also responding to these modifications [31]. This effect has been well documented in ant swarms, in which ants lay down pheromones as they walk that, in turn, attract subsequent ants [32]. An analogous stigmeric effect is achieved by "swarms" of humans that make a terrain more attractive to others by creating paths with their own steps—exactly the kind of path-formation phenomena that we will experimentally explore. Stigmergy has recently been proposed as an important mechanism for achieving multirobot cooperation [33] and robustly interacting software systems [34]. For example, robots have been shown to be capable of creating large-scale architectural structures even without direct communication, as long as each robot can both effect their environment and respond to the effects made by other robots.

V. Trail Formation

Human groups' paths will later be compared to an agent-based computational model from biophysics [1], but it is also helpful to compare people's paths to optimal accounts as well. One optimal account is provided by the mathematics of "minimal Steiner trees (MSTs)." An MST is the set of paths that connects a set of points (e.g., destinations) using the minimal amount of total path length. If we restrict ourselves to only creating direct connections between destination points, then the shortest total-path network that connects a set of points is called the minimal spanning tree. Fig. 1 presents two examples of minimal spanning trees for configurations of three and four points. However, if we allow new points, called Steiner points, to be created to reduce the total path length, then we can often have a shorter path than the minimal spanning tree. In the 1600s, Fermat and Toricelli independently discovered methods for constructing MSTs for

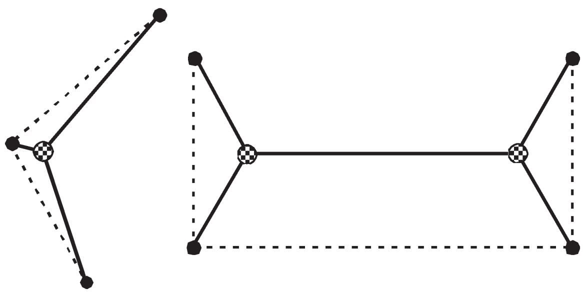

Fig. 1. Examples of minimal spanning trees (dashed lines) and MSTs (solid lines) connecting arrangements of three and four black points. The Steiner points, added to the configurations to reduce the total path length connecting the points, are shown as checkered dots.

three points. They showed that there are always three lines coming out of a Steiner point, and each pair of adjacent lines establishes a 120° angle.

Fig. 1 also shows the MSTs for the configurations of points. For the rectangular arrangement of points, the MST is interesting because it demonstrates that Steiner points can connect to other Steiner points, but they still only have three lines separated by 120° angles. For a more complex set of points, the MST still consists only of connections among original points and added Steiner points that always are connected by three 120° angles. Finding MSTs is a notorious NP-complete problem, with all known, provably optimal algorithms requiring an exponential increase in computation as the number of destination points increases linearly [35]. However, analog devices, such as soap films, have been shown to frequently create MSTs [36]. If pins are driven through two plastic parallel plates and the device is dipped in a soap bath, the resulting film when the device is removed will often form an MST due to surface-tension minimization. The existence proof of soap film MSTs leaves open the possibility that other complex systems that operate according to different principles, including groups of people, might also approximate MSTs. The determination of MSTs is not only a mathematical curiosity but is practically important for laying out airports, integrated circuits, water pipes, railroad tracks, or any other network in which travel time or path material are valuable commodities.

Although several algorithms in computer science exist for finding decent approximations to MSTs [37], it is also important to know whether and when decentralized systems like groups of ants [38] or people can approximate MSTs. Prior work has shown that individuals can approximate MST solutions [39] and optimal solutions to the related Traveling-Salesperson Problem [40]. However, our current paper explores the degree to which entire groups of people approximate MST solutions when these solutions are not explicitly requested but may emerge from participants reusing paths created by predecessors.

To study group path formation in people, we adapted a software platform that we have developed to conduct other multiparticipant experiments over the Internet [28], [41]. The software allows us to have moderate group sizes of up to 80 people interacting simultaneously in a virtual environment, viewing the moment-by-moment behaviors of themselves and the other participants. Using this software, we developed a virtual environment in which people move between destinations and are motivated to take advantage of each others' paths.

VI. Method

Three hundred and two undergraduate students from Indiana University served as participants in order to fulfill a course requirement. The students were run in 34 groups with 6–12 participants per group. The groups received one of two different sets of stimuli.

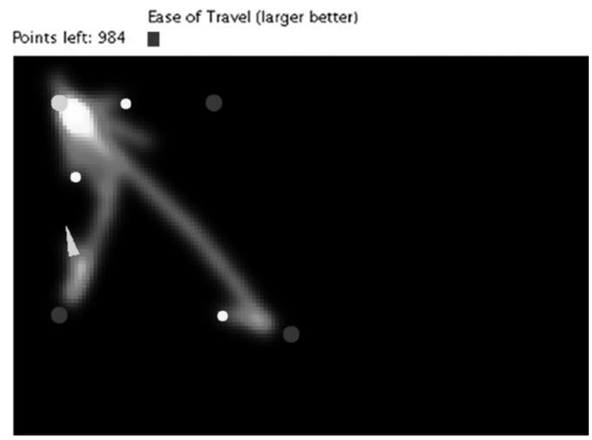

Fig. 2. Sample screen display during an experiment with four participants. Participants see themselves as green triangles (shown above as light gray). All of the other participants appear as yellow dots (appearing above as light gray). Cities are shown as blue dots (shown as dark gray), except for the participant's destination city, which appears in green (shown as light gray). The cost of every screen location is coded by its brightness using a red palette, with brighter locations signifying lower costs.

The participants were all physically located in the same room for experimental control. The participants were instructed that their task was to travel from city to city while earning the most points possible. Each participant's point total was shown in the upper-left corner of the screen. They were instructed that they would receive a certain number of points for every destination city reached but that they would lose points for every step taken. They were also instructed that each cell of the 150 $\times$ 100 world would have its own changing cost and that a cell's cost decreased as a function of the number of times it had been trodden. The instantaneous cost of a cell is shown by its brightness, with more expensive cells appearing darker, as shown in Fig. 2.

Each experiment was divided into 5-min periods of travel across six configurations chosen from the eight configurations in Fig. 3. Each configuration consisted of three to four cities, shown as large blue dots. At any time, each participant was randomly assigned one of the cities as their destination, and this destination was colored green. When participants reached their destination city, a new and different destination city was selected, and the participant was given a number of points equal to 15 times the distance between their starting and destination cities. Participants began each configuration at uniformly distributed random locations.

Across both groups of participants, eight different configurations of cities were tested. The coordinates of the points for each configuration are expressed as $\{X, Y\}$ pairs and were as follows:

Equilateral triangle: {50, 10}, {10, 79}, {9, 79}
Isosceles triangle: {50, 10}, {30, 89}, {70, 89}
Canonical obtuse triangle: {10, 63}, {50, 50}, {106, 63}
Rotated obtuse triangle: {10, 10}, {50, 50}, {90, 60}
Large rectangle:{10, 10}, {140, 50}, {140, 10}, {10, 50}
Small rectangle: {10, 10}, {70, 30}, {70, 10}, {10, 30}
Rotated isosceles: {80, 16}, {21, 50}, {57, 80}
Rotated isosceles with Steiner point: {80, 16}, {21, 50}, {57, 80}, {48, 55}.

Participants appeared as yellow dots, except for a participant's own position, which appeared on the screen as a green triangle, with the most acute corner of the triangle denoting the participant's heading direction. The heading direction was altered by pressing the computer keyboard's right arrow key to move clockwise by 3° and the left arrow to move counterclockwise by 3°. This method of specifying motions does incur a time cost to rotation, but if participants held down the

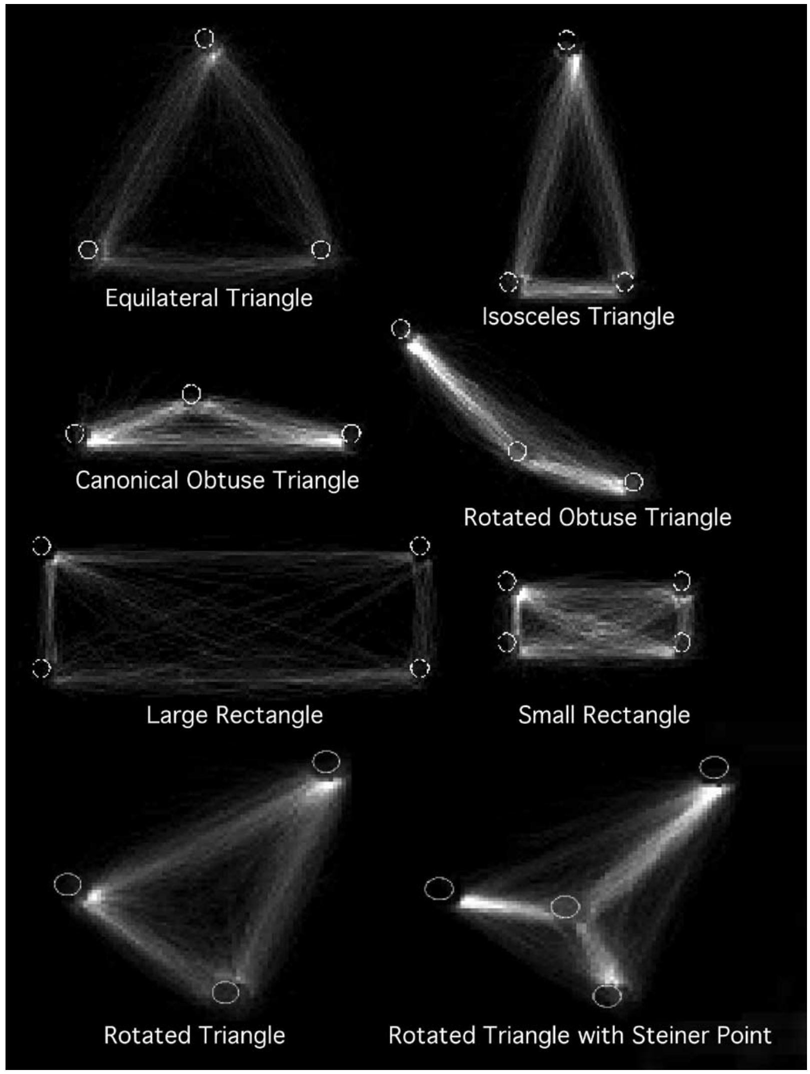

Fig. 3. Cumulative steps taken on each cell of the eight configurations. The brightness of a location is proportional to the number of times that it was stepped on by all participants. The destinations are indicated by white circles.

left or right arrow keys, they would rotate at a relatively rapid rate of $340°/\mathrm{s}$. Pilot experiments indicated that alternative methods of indicating routes, such as specifying absolute locations with a mouse, had the disadvantage of not engaging participants to make moment-by-moment assessments and adjustments of their routes. Participants moved forward by pressing the up arrow key and kept moving until they pressed the down arrow key. When participants reached their destination, they automatically stopped and need to press the up arrow to once again move forward.

Every step taken by every participant influenced the travel cost of each cell of the world. While participants' steps decreased travel cost, travel cost increased over time to a maximum value to model the regrowth of obstructions or the erosion of path systems. Travel ease for each cell gradually decreased by $N\%$ every second, where N is the number of participants. Tying path erosion to the number of participants roughly equated the overall travel ease for groups of different size. When a participant stepped on a cell, the cost decrease was diffused to neighboring squares according to a Gaussian distribution. Specifically, the ease of travel of every cell was increased by $C_1 \exp(-C_2 D_{P,L})$, where $D_{P,L}$ is the Euclidean distance between participant P and a cell's location L, C_1 is a parameter controlling the influence of a participant on a cells' travel cost, and C_2 is a parameter controlling the extent of diffusion to neighbors. For our experiments, C_1 was 0.007, and C_2 was 0.18. Fig. 2 shows how the moment-by-moment changes to cells' travel costs were reflected by the brightness of the cells.

Participants were shown movement updates for another participant every time the other participant turned or every 2 s, whichever occurred first. Participants also received continual updates on their score and the instantaneous cost of stepping on their current cell. The instantaneous cost was shown graphically by the length of a blue bar at the top of the screen.

VII. Results

Fig. 3 presents the results from the experiment in an intuitive manner that will subsequently be refined for statistical analysis. For each of the eight configurations of cities, the number of times every location was stepped on is indicated by its brightness. The locations of the cities are indicated by circles. Configurations that were designed to be compared to one another are placed side by side in Fig. 3. One immediately apparent result is that participants do not generally create MSTs. However, there are deviations from beeline paths between cities, and frequently, these deviations are in the direction toward

MST paths. In what follows, such deviations will be referred to as "pro-Steiner deviations." There was apparently more pro-Steiner deviation from beeline pathways for the isosceles than for the equilateral triangle. This is indicated by the large bright area directly below the top city for the isosceles but not for the equilateral triangle.

For the second pair of configurations, the same obtuse 150° triangle is presented in two orientations. In the canonical obtuse configuration, the longest path is a horizontal line. For the rotated obtuse, the longest path has an undistinguished orientation of 122°. For both configurations of points, the MST is identical to the minimal spanning tree and is simply the combination of the two short paths. Fig. 3 shows that there is more travel close to the beeline path connecting the furthest cities for the canonical than the rotated obtuse.

For the third pair of configurations, two rectangular configurations of cities have identical aspect ratios, differing only in their scaling factor. Fig. 3 shows more deviation of the outer paths toward the MST for the small than for the large rectangle.

The final pair of configurations compares an isosceles triangle to the same isosceles triangle with its Steiner point included as a destination point. When the Steiner point is added to the triangle, all of the other paths deviate more toward the Steiner point. In fact, for this path system, people do frequently use the MST path to reach any pair of cities, which is why the outer beeline paths are so dim.

The plots in Fig. 3 are imprecise because they do not distinguish between which pair of cities a person was traveling. For our statistical analysis, we considered each participant's journey from one city to another. For each step on this path, we measured the distance of the step to the beeline path and coded it as positive if the deviation was in the direction of the MST path and negative if it was in the opposite direction. If a participant ever traveled a distance more than 20 cells from the relevant beeline path, then their entire journey between the two cities was ignored. This was needed because a participant would rarely decide to travel outside of the configuration of destinations to engage in task-irrelevant behaviors (e.g., making spirals or tracing the edges of the world). Fewer than 1% of the routes were thus eliminated. Otherwise, the scalar measure of a participant's beeline path deviation is an integration of the step-by-step deviations.

Graphical representations of this quantitative measure of deviation from beeline paths are shown in Fig. 4, and the summary is shown in Table I. To generate Fig. 4, a spline is plotted to connect the two traveled cities via a point in the middle that reflects the integrated deviation. The spline should not be interpreted as the exact path. Only the maximal extent of deviation of a curved line from the beeline path is meaningful. This technique has the added advantage of allowing us to look at asymmetries, where people take different paths from A to B than from B to A. The arrows on a path indicate its directionality.

Using this measure of integrated deviation, results confirm that the isosceles triangle is significantly more pro-Steiner (i.e., larger deviations in the direction of the MST) than the equilateral triangle paired two-tailed T-test $t(297) = 3.32$, $p < 0.001$. The isosceles triangle also had a significant asymmetry for paths, with the paths to the top point being significantly more pro-Steiner than the paths from the top point $t(46) = 2.76$, $p < 0.01$. Table II quantifies this asymmetry by showing the average difference between bottom-to-top paths and their paired top-to-bottom paths. A standard convention is adopted by which positive differences in Table II signify greater pro-Steiner deviation for the path that is closer to the MST at its end rather than its beginning.

For the obtuse triangles, the longest path is more pro-Steiner for the rotated than canonical triangle $t(113) = 4.47, p < 0.001$. Conversely, for the shorter paths, the canonical triangle deviates more from the beeline path than the rotated triangle $t(211) = 7.50, p < 0.001$. These effects cancel each other out, resulting in the two triangles having approximately equal average deviations (shown in Table II). In addition,

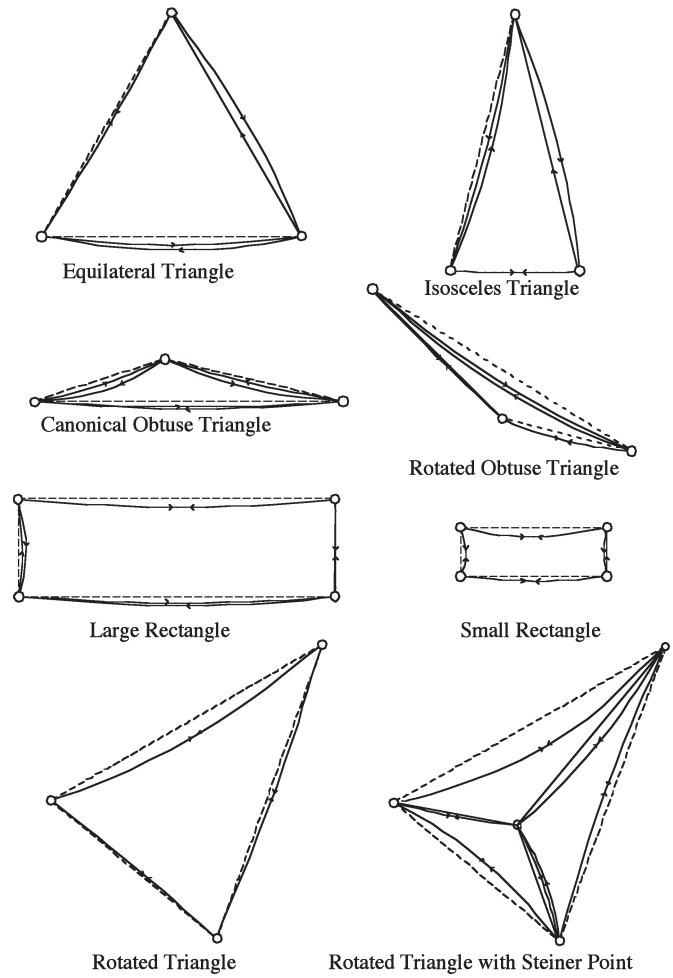

Fig. 4. Deviations of participants' actual paths from beeline paths (shown by dashed lines) for eight configurations of cities. The arrows indicate the traveled direction and maximal displacement of the paths from the beeline. Solid line paths are proportional to the integrated deviation of the participants' steps.

the canonical triangle has significant path asymmetries, with paths to the upper central point deviating significantly more from the beeline path than paths going from the upper point $t(55) = 3.20$, $p < 0.01$. The rotated triangle did not show any asymmetry.

The small rectangle was significantly more pro-Steiner than the large rectangle $t(283) = 2.40$, $p < 0.02$, but neither rectangle had significant path asymmetries. Finally, only considering the outer paths, the rotated isosceles triangle with the Steiner point is significantly more pro-Steiner than the rotated isosceles triangle $t(264) = 2.67$, $p < 0.001$, but neither triangle showed significant asymmetries. Consider all five of the nonequilateral-triangle configurations, we find that the two longer beeline paths forming the smallest angle have more pro-Steiner deviations than the third, shorter path $t(264) = 3.1$, $p < 0.001$. This is indicated in Table II by the positive values for the asymmetries.

To validate the strategies suggested by the participants' pathways, we explicitly asked two groups of participants to describe their route-following strategies. We ran a classroom demonstration of the experiment to two groups with 18 and 22 students. After using the upper six configurations shown in Fig. 3, we gave each of the participants an open-ended questionnaire followed by a multiple-choice question. For the open-ended questionnaire, we instructed participants, "When you were deciding what route you would take to get to your destination, describe some strategies that you used to guide your route."

TABLE I
AVERAGE DEVIATIONS (DISTANCE IN CELLS) FROM BEELINE PATHWAYS AND SYSTEMATIC ASYMMETRIES (AVERAGE DISTANCE IN CELLS) BETWEEN X-TO-Y AND Y-TO-X PATHWAYS

Configuration	Deviations from Bee-Line paths	Systematic Asymmetries
Equilateral Triangle	-1.30	N.A.
Isosceles Triangle	+0.01	+2.22
Canonical Obtuse Triangle	+0.77	+1.92
Rotated Obtuse Triangle	+0.75	+0.08
Large Rectangle	+0.16	N.A.
Small Rectangle	+1.19	N.A.
Rotated Triangle	+1.17	+0.16
Rotated Triangle with Steiner Point	+3.30	+0.66

Notes: Positive deviations indicate deviations away from beeline paths in the direction of the MST path. Negative deviations indicate deviations away from MST paths. The asymmetries are calculated by considering only routes where the X-to-Y path is configurationally distinguishable from the Y-to-X path. Positive asymmetries indicate greater deviation toward the MST path later in the route than earlier. Negative asymmetries indicate greater deviation toward the MST path earlier in the route than later.

TABLE II
STRATEGIES REPORTED TO OPEN-ENDED AND MULTIPLE-CHOICE QUESTIONNAIRES

Strategy	Percent Reported	
	Open ended (%)	Multiple choice (%)
Bee-Line Paths	13	5
Most Used Paths	16	8
Compromise	65	75
Drafting	6	4
Planning for the Future	3	6
Other	10	2

Note: Responses to the open-ended question sum to more than 100% because participants occasionally reported using more than one strategy.

Participants were given 4 min to respond. Following this, they were given the following multiple-choice question:

Which of the statements below best describes the strategy that you used most often to determine your route to destinations?

1) I went along the most direct route to the destination (Beeline paths).
2) I went along the routes that were most well traveled (Most-used paths).
3) I compromised between using the most direct routes and the most traveled routes (Compromise).
4) I waited until one or more people traveled in the direction I wanted to go, and then I followed them (Drafting).
5) I tried to create routes that might be indirect paths to my destination but would create paths that would be helpful for myself or the group later (Planning for the Future).
6) Others.

These six response items were culled from previous postexperimental interviews with participants and represent the strategies that they most often described. The responses to the questionnaires, shown in Table II, indicate that the most frequently described strategy was a compromise between using beeline paths and the most traveled routes. Interestingly, very few participants spontaneously reported creating pathways that would be useful for future travels (e.g., the "Planning for the future" strategy).

We explored the possibility of group size modulating the degree of pro-Steiner deviation. We did not find any evidence for this, presumably because groups were approximately equated, despite size differences, because the path erosion rate was based on the number of participants. Also, participants could occupy the same location, thus minimizing the influence of traffic congestion.

VIII. DISCUSSION

The experiment revealed several interesting trends in the groups' paths. First, the greater pro-Steiner deviation for the isosceles than the equilateral triangle is interesting, because it can be proven that the largest total savings of path distance for the MST over the spanning tree or the set of beeline paths is found for the equilateral triangle and not the isosceles triangle. So, deviations toward MST would have been much more effective for the equilateral triangle, but they were not found. Having a particularly advantageous optimal path is no guarantee that a group will find it. For the equilateral triangle, there is little incentive for a pioneer to move straight down from the top city; the terrain is dark and costly there (see Fig. 2). For the isosceles triangle, this area becomes brighter because of the overlapping diffusion from the two beeline paths from the top city, and once people go straight down, it is attractive for others, and so the vertical path is extended still further.

Second, people seem to be especially attracted to the canonical horizontal path for the obtuse triangle, even though all directions are equally easy to specify. This is indicated by long paths that stay closer to the horizontal path for the canonical than the rotated obtuse triangle and by greater deviation of the shorter paths toward the long path for the canonical than rotated obtuse triangle. Prior research has supported a privileged status for horizontal and vertical axes [42]–[44]. It is plausible that participants gravitate toward these canonical axes because of their status as privileged frames of reference.

More surprisingly, participants occasionally took different routes going from A to B than from B to A. Many of these asymmetries can be explained by participants being more willing to deviate from beeline paths at the end of a journey than at the beginning. For example, both of the paths to the top point of the isosceles triangle in Fig. 4 are more pro-Steiner than the paths going away from the top point.

This is predicted if people are more likely to be attracted by the low-cost region underneath the top point (see Fig. 3) on their way to the top point. On the way from the point, people may not want to take an immediate detour to take advantage of the low-cost region. Taking detours away from beeline paths and toward low-cost paths late, rather than early, during trail forging also explains the asymmetries found in the obtuse triangles. This heuristic is an extension of the "road climbing" heuristic of Bailenson *et al.* [45], [46], by which there is a preference for long and straight routes in the local area containing the origin, as opposed to destination, of a trail. Their heuristic, also proposed to explain asymmetries in route choices, shares with our proposal the notion that people are particularly motivated to make early progress toward their destinations.

Another surprising result was the greater pro-Steiner deviation from beeline paths for the small, rather than the large, rectangle. A similar explanation, instantiated by the computational model that follows, may be at work here as for the isosceles triangle. Pronounced deviations from beeline paths are observed when two paths are close enough that a path created for one purpose is available to be used for another purpose. This kind of path reuse results in paths that increasingly blend into one another over time.

A final strong case for path reuse comes from the greater pro-Steiner deviation for the rotated triangle when the Steiner point was added as a destination. Early steps along the paths involving the Steiner point city are later exploited by participants traveling among the outer cities. This, in turn, makes the outer paths still dimmer, prompting even greater use of the MST path.

IX. Computational Model of Path Formation

Clearly, the optimal MST account does not provide a faithful model of our results. However, a computational model developed by Helbing *et al.* ([1]; see also [47] and [48]) has assumptions that are highly consistent with our experimental protocol and does a good job of accounting for our major results. This agent-based "Active Walker" model assumes walkers move to destinations, and as they take steps, they affect their environment, facilitating travel for subsequent walkers. Walkers compromise between taking the shortest way to their destination and using existing strong trails. The core idea is that each spot on a terrain has a potential function to describe its ease of travel,[1] where the ease of travel of a spot is increased by walkers' steps on it and decreases as the path erodes. The ease of travel of position $\mathbf{r}$ at time t is expressed by $G(\mathbf{r}, t)$, and its change is governed by

$$\frac{\mathrm{dG}(\mathbf{r},t)}{\mathrm{d}t} = \frac{1}{T(\mathbf{r})}\left[\mathrm{G}_0(\mathbf{r}) - \mathrm{G}(\mathbf{r},t)\right] + I(\mathbf{r})\left[1 - \frac{\mathrm{G}(\mathbf{r},t)}{\mathrm{G}_{\max}(\mathbf{r})}\right]\sum_{\alpha}\delta\left(\mathbf{r} - \mathbf{r}_{\alpha}(t)\right) \quad (1)$$

where the δ function is Dirac's delta function that only is positive for values of 0. $T(\mathbf{r})$ is a parameter reflecting the durability of trails from erosion, and $I(\mathbf{r})$ reflects the influence of steps on changing the ease of travel. The first factor in (1) decreases ease of travel for a location based on how much easier travel is for a location compared to its baseline ease of $G_0(\mathbf{r})$, which is 0 in our simulations. The second factor increases ease of travel as a function of the trodden-upon location's current ease compared to its maximally possible ease $G_{\max}(\mathbf{r})$. The attractiveness of a trail segment at location $\mathbf{r}$ from the perspective of an agent at $\mathbf{r}_{\boldsymbol{\alpha}}$ is formalized as a potential function and is based on its proximity to agent α and an integrated spatial average of the locations' individual comfort levels

$$V_{\mathrm{tr}}(\mathbf{r}_{\alpha}, t) = \int \mathrm{d}^2 r e^{\frac{-|r - r_{\alpha}|}{\sigma(r_{\alpha})}} G(\mathbf{r}, t). \quad (2)$$

Thus, the trail potential will be influenced by every location's ease of travel, but these influences drop off as an exponential function of their distance to an agent. The $\sigma(\mathbf{r}_{\boldsymbol{\alpha}})$ term reflects agent α's visibility—the extent to which it is influenced by distant cells. As visibility increases, the agent will make increasingly large deviations away from beeline paths, in order to take advantage of easy-to-travel locations. Finally, the walking direction e_{α} of agent α is a linear combination of the destination and the trail potential, divided by a factor that normalizes the direction

$$e_{\alpha}(\mathbf{r}_{\alpha}, t) = \frac{\mathrm{d}_{\alpha} - \mathbf{r}_{\alpha} + \nabla_{\mathrm{r}\alpha}\mathrm{V}_{\mathrm{tr}}(\mathbf{r}_{\alpha}, t)}{|\mathrm{d}_{\alpha} - \mathbf{r}_{\alpha} + \nabla_{\mathrm{r}\alpha}\mathrm{V}_{\mathrm{tr}}(\mathbf{r}_{\alpha}, t)|}. \quad (3)$$

Before any trails have been formed, walkers will simply take beeline paths to their destinations. However, as trails begin to be formed, walkers will often take detours to take advantage of the increased comfort of the trails, thereby further reinforcing the comfortable trails.

Varying only the single parameter $\sigma(\mathbf{r}_{\boldsymbol{\alpha}})$ that controls the visibility (hence influence) of paths, the model is able to broadly predict several aspects of our results. Sample screen plots from the model are shown in Fig. 5. Consistent with our experimental results, the model does not generally predict MST trails but, rather, trails that deviate away from beeline paths toward MST paths. The model correctly predicts more pro-Steiner deviation for the isosceles than for the equilateral triangle. Moreover, for the isosceles triangle, the model correctly predicts that the greatest deviation from beeline paths is found near the top triangle point (compare to Fig. 3). It also correctly predicts greater pro-Steiner deviation with passing time. Over time, the originally separated beeline routes become progressively zipped together, starting at the top point.

The model correctly predicts greater pro-Steiner deviation from beeline paths for small than large rectangles for the same reason it predicts greater deviations for the isosceles than for equilateral triangle. Paths are closer to one another for the small than for the large rectangle, and, consequently, there is more opportunity for reuse of close paths.

The model correctly predicts greater pro-Steiner deviation from beeline paths when the Steiner point is added to an isosceles triangle, because the paths to the Steiner point become attractive candidates for reuse by walkers going to destinations on the outer triangle.

While the model predicts the general appearance of the obtuse-triangle trails, it is rotationally invariant. Consequently, the original model does not predict differences in trails for the canonical and rotated obtuse triangles. However, a small change to the model suffices to accommodate our empirical results. If a generalized Minkowski r-parameterized distance function is used to calculate the distance between a walker and a location (instead of the standard Euclidean distance function in which $r = 2$), and if $1 \leq r < 2$, then the walker will be more strongly influenced by horizontal and vertical trails than diagonal trails.

Finally, the model can predict route asymmetries, but they are in the opposite direction from our results. For example, in Fig. 5, for the plot labeled "Isosceles Triangle with Decreased Path Visibility," there are slightly different paths connecting the light gray and dark gray cities, as well as the light gray and medium gray cities. The walker going to the light gray city from the dark gray city is taking the outer path, while the walker going from the light gray city to the dark gray city is following the inner path. This is the opposite of our results, where we found

[1]Our conceptualization of the "Active Walker" model is somewhat different from that of Helbing *et al.* and is tailored to our experiment. While we frame the model in terms of trail erosion and ease of travel, they frame their model in terms of obstructing vegetation regrowth and comfort level. However, the equations are identical.

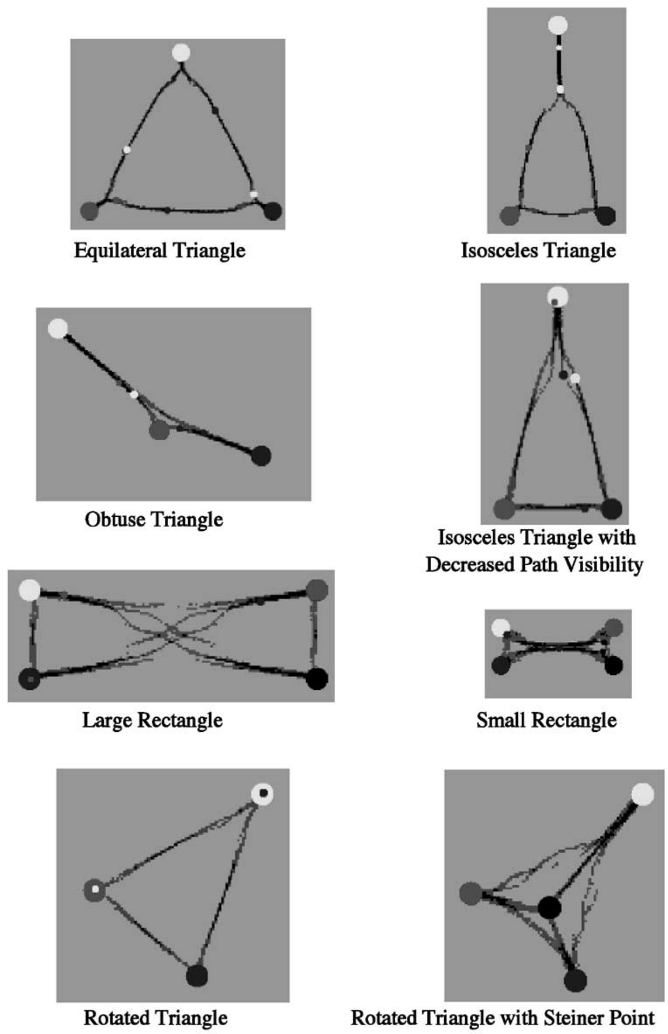

Fig. 5. Sample screen plots of the Active Walker model of Helbing *et al.* (1997) for the destination configurations used in our experiment. Destinations are shown as large circles and individuals are shown as small dots whose shade matches the shade of their destination city. The darkness of a cell is positively related to its level of comfort for walking.

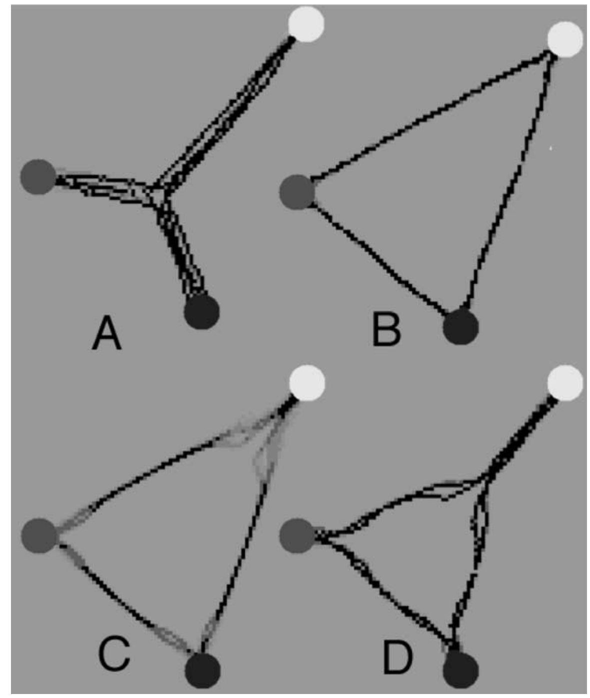

Fig. 6. Influence of parameters on path formation within the Active Walker model. The darkness of cells is positively related to their level of comfort of walking and indicate the eventual paths after 1000 iterations. (a) $T = 0.1$, $I = 1000$, $\sigma = 100$. (b) $T = 0.1$, $I = 1000$, $\sigma = 1$. (c) $T = 0.001$, $I = 10$, $\sigma = 10$. (d) $T = 0.1$, $I = 1000$, $\sigma = 10$.

greater pro-Steiner deviation going from the dark gray (or medium gray) to the light gray city than vice versa. One relatively simple way of modifying the Active Walker model that is consistent with the "road climbing" heuristic of Bailenson *et al.* [45] and our results would be to dynamically alter the path-visibility parameter over the course of a journey from one city to another. By linearly increasing path visibility across each intercity journey, walkers would start out their journeys taking the most direct beeline path toward their destination and then gradually become more attracted by low-cost detours.

X. Implications for Concrete and Abstract Path Design

We must be careful not to overgeneralize the results from the current experiment to more naturalistic concrete, let alone abstract, path-formation situations. People spontaneously creating paths do not always have the group-level survey that our participants did. Furthermore, our participants used computer controls rather than their legs to form the paths, and the paths emerged on the order of minutes rather than years. Bearing these caveats in mind, some cautious implications can still be drawn. First, there are specific applications of our results to those concerned with spontaneous communal-path formation. Architects, park designers, and city planners may be able to use our results to infer likely biases in people's navigational strategies and the consequences of these strategies when people are influenced by the existing pathways. For example, we observed that people are more likely to deviate from beeline pathways to take advantage of low-cost pathways late, rather than early, in their route. This information can be used to predict traffic-flow patterns and to strategically position attractions. Knowing that scale may affect the deviation of formed paths from beeline paths could dissuade engineers from assuming that path formation observed in their toy models will correctly scale up to the real-world full-scale system.

The success of the Active Walker model suggests an alternative method of crowd control. The most common method of crowd control is through direct order or laws. If we wish to direct pedestrian traffic, for example, we may institute rules or physical barriers that prohibit certain movements. The cost of such prohibitions is decreased pedestrian morale and the perception of excluded possibilities [49]. An alternative method of crowd control is by changing the structure of the environment such that certain navigational behaviors are facilitated, whereas others are indirectly hindered. Even without instituting physical or abstract barriers, it may be possible to indirectly control collective behavior with substantial efficacy. Collective behavior is potentially more controllable than isolated individual behavior because of the strong influences among the individuals' behavior. A small pressure can often be magnified by the positive feedback involved in individuals following other individuals [31].

Under this new approach toward fostering collective organization, the aim would be to facilitate the development of self-organized patterns rather than dictate high-level structures via top–down control. To this end, parametric variation within the Active Walker model can be used to guide policy decisions. Fig. 6 shows four simulations of the Active Walker model when provided the same arrangement of destinations but with four different configurations of parameter values.

The paths produced by Fig. 6(a) and (b) differ only in the σ parameter. When σ is high (Panel A), then the visibility/influence of the existing paths is large [see (2)], and agents will be strongly biased towards reusing previously formed paths. This produces a path system that closely approximates a MST. When path visibility is low, then agents' paths deviate very little from beeline paths. The overall strength of pathways is held constant between Panels C and D but is achieved by different mechanisms. In Panel C, paths are fairly permanent [T is low, see (1)], but each individual step has only a small effect on pathways (I is low). By contrast, in Panel D, paths rapidly degrade once made (T is high), but each step has a large influence on the environment (I is high). Clearly, there are more deviations toward the MST path for Panel D than for Panel C. This is because rapid decay of paths allows the group to create new paths without being strongly constrained by the original beeline paths. The important point from Fig. 6 is that the Active Walker suggests specific interventions, depending on the planners' goals. For situations where conserving the total amount of pathway is desirable (e.g., when vegetation must be cut down to create the paths), planners should explore ways of increasing path visibility, the efficacy of steps, or path decay. Given the empirical success of the Active Walker model with our groups of people, varying its parameters becomes a potentially useful way of not only predicting, but also controlling, spontaneous paths.

This conceptualization of design planning as facilitating self-organization rather than dictating final form may have an important moral for abstract and not only geospatial path systems. There is a growing interest in self-organized agent systems in computer science, particularly those that develop coordinated structures by having the agents change their virtual environment and respond to changes produced by others—a virtual form of stigmery [50]. Swarms of computer agents have been shown capable of producing good solutions to optimization problems [31] and demonstrating impressive levels of the fault tolerance and flexibility [33]. The abstract design principle underlying our current experiment and modeling is: Globally efficient network structures can arise from local interactions among agents, each of which is positively influenced by the others. Beyond this generalization, the specific parametric interventions in the Active Walker model are likely to be useful for constructing efficient path systems in peer-to-peer, distribution, and information-sharing networks. In particular, efficient information passing may be achieved by gradually increasing the bandwidth of already popular network paths. Within this framework, Fig. 6 suggests that incorporating high levels of path decay and visibility will allow the agents to spontaneously converge upon networks with short total path length. The Active Walker model, and its empirical support, provide some grounds for optimism that distributed agents acting locally can establish efficient networks.

XI. Conclusion

A complex adaptive-systems perspective may be an exciting perspective for exploring group behavior [3]. As an addition to the social psychologists' tool kit, it can provide accounts for why structures such as road networks have the appearance they do. Our current results indicate that people do indeed take advantage of trails left by others.

To a first approximation, our group behaviors are well modeled by the Active Walker model from biophysics. The model can account for differences in scales and topologies of destinations, the influence of time on emerging paths, and the approximate distribution of actual steps. There were some discrepancies—the direction of asymmetries and rotation invariance—but minor model modifications can correct these mispredictions. The essential insight of the model seems to fit our groups well: People's movements are a compromise between going where they want to go and going where others have gone.

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